# MODELS OF THE ACTUARIAL BALANCE OF THE PAY-AS-YOU-GO PENSION SYSTEM. A REVIEW AND SOME LESSONS[*]

**María del Carmen Boado-Penas†, Junichi Sakamoto‡ and Carlos Vidal-Meliá♠.**

**17 September 2009**

**ABSTRACT**

This paper reviews the two main methods used by government Social Security departments to draw up the so-called actuarial balance of the pay-as-you-go pension system, focusing especially on their results, methodology and actuarial issues. The specific models studied are those in Sweden, the United States and Japan. The authors suggest that it would be interesting and potentially productive politically for official information on these two types of actuarial balance to be provided on an annual basis for all public pension systems in order to improve their transparency, credibility and solvency. In this respect ISSA, the World Bank, the OECD and other international organisations (for example, the Pension Benefits and Social Security (PBSS) section of the International Actuarial Association) could be supportive in developing and enforcing international accounting and actuarial valuation standards for pay-as-you-go pension systems.

(**JEL**: H55, J26, M49).

**Key words**: Japan, Notional accounts, Public pensions, Retirement, Sustainability, Sweden, USA.

---

[*] Carlos Vidal-Meliá is grateful for the financial assistance received from the Spanish Ministry of Education and Science (Ministerio de Educación y Ciencia) project SEJ2006-05051. María del Carmen Boado-Penas thanks the Department of Education, Universities and Research of the Government of the Basque Country (IT 313-07). We would also like to thank Ole Settergren for his invaluable help and comments and Peter Hall for his English support. Preliminary versions of this paper were presented at the 3rd Workshop on Risk Management and Insurance Research in Madrid (Spain) and at the XI Spanish-Italian Congress of Financial and Actuarial Mathematics in Badajoz (Spain). Any errors are entirely due to the authors. The opinions expressed in this paper are the authors' and do not necessarily represent the views of the University of Valencia, the University of the Basque Country or the Nomura Research Institute.

† Department of Economics. Keele Management School, Keele University, Keele (England). (e-mail: m.d.boado-penas@econ.keele.ac.uk).

‡ Nomura Research Institute, 1-6-5 Marunouchi, Chiyoda-ku, Tokyo 100-0005 (Japan) (e-mail: j-sakamoto@nri.co.jp )

♠ (Corresponding author) Department of Financial Economics and Actuarial Science, University of Valencia, Avenida de los Naranjos, s.n. 46022 Valencia. (Spain). (e-mail: carlos.vidal@uv.es).

## 1.-INTRODUCTION

According to Settergren (2008), one possibly useful way of reaching the objective of better designed and managed public pension systems would be to provide better information on the financial development and status of those systems. In many countries, reporting on these aspects in public pay-as-you-go (PAYG) pension systems is not fully established. This reduces the accumulation of knowledge as to how the pension system interacts with the demography and economy of these countries, reduces informed public debate on public pensions, increases the longevity of old misunderstandings and is fertile ground for new ones. For Bovenberg (2003), transparency about objectives and risk-sharing agreement is a key requirement for all pension systems. Explicit agreements about how to share risks become even more important as ageing makes systems more vulnerable to shocks such as unexpected longer longevity or inflation. Being explicit about the risk-sharing agreement by setting clear rules and having more transparent accounting increases the costs associated with breaking the intergenerational contract and thereby reduces the political risks surrounding public PAYG pension systems.

The so-called actuarial balance of PAYG pension systems, Boado-Penas (2008), provides a suitable answer to all these issues and also supplies a positive incentive to improve financial management by eliminating or at least minimising the traditional mismatch between the planning horizons of electors and politicians, often only four years, and the system itself. As we will see later, the minimum time horizon should be the system's turnover duration, the normal value of which varies between 30 and 35 years, although actuarial reports often contemplate a minimum horizon of 75 years.

This paper looks mainly at what will be described as the actuarial balance of the PAYG system, with particular attention given to the Swedish and US models, although we will also comment on some of the more significant aspects of the Japanese model[1].

According to the BOT (2009), a very detailed annual actuarial balance - an actuarial report, in fact - has been compiled in the United States since 1941, and from 2002 it has included stochastic methodology. The so-called "US" actuarial balance drawn up by US Social Security - similar to that published by the authorities in Japan every five years (Sakamoto (2005)) and in Canada every three years since 1966 ((OSFIC) (2007 and 2008)) - is not a balance sheet in the traditional accounting sense of the term, with a list of assets and liabilities which consider an indefinite horizon.

Compiling an official actuarial balance sheet has been normal practice in Sweden since 2001. This actuarial balance sheet - in the form it takes in Sweden - has until now attracted little attention from academics. This is surprising given that in the literature there are a great many methodologies applied to analyse the viability or sustainability of pension systems or to forecast aggregate spending, and this field is of special interest to a number of researchers. As far as we are aware, only in the cases of Japan[2] and Spain[3] has the actuarial balance sheet with its typical structure of assets and liabilities been used by researchers, and on an official level it has not been used outside Sweden[4].

According to Vidal-Melia et al (2009a) the most commonly used methodologies for making aggregate projections of spending on pensions or for analysing the sustainability of pension systems are:

---

[1] Due to lack of space, specific results of the latest actuarial valuation for Japan will not be presented here, although some of the system's more interesting differences will be commented on. See AAD (2006).

[2] Takayama (2005) uses the actuarial balance sheet as an element to analyse proposals for reforming the pension system, although the list he presents of the items it comprises is not very developed.

[3] Boado et al. (2008) compile an actuarial balance sheet to assess the solvency of the Spanish system and compare it to the Swedish system. They also develop the concepts of the contribution asset and the average turnover duration for defined benefit PAYG systems. Vidal-Meliá et al (2009a) use the balance sheet of the Spanish system to support the introduction of an automatic balance mechanism (ABM).

[4] At the moment, no other country uses the actuarial balance sheet in the form it takes in Sweden, but some countries do make financial or actuarial projections, and the analysis is very similar to that done in the USA, Canada or Japan. On this aspect see the papers by Lefebvre (2007) and TEPC (2007).

1) Aggregate or growth accounting models: The aggregate accounting approach relies on making a variety of assumptions regarding the economy as a whole, taking into account future trends in demography (fertility rates, migration flows and life expectancy), economic conditions (participation and employment rates, productivity, wages and interest rates) and institutional factors (coverage and pension levels). These are used mainly for making aggregate projections of spending on pensions. Despite the fact that these models are becoming more and more complex as they are made heterogeneous, their main advantage is that they are easy to apply and accurately reproduce the reality of the pension system. They are often referred to by some authors as actuarial models. They are also frequently used by public authorities and organisations; the Ageing Working Group - the technical working group of the European Union's TEPC which is responsible for spending forecasts - follows this basically deterministic approach, although not all the countries involved apply it.

2) Micro-simulation models. The working lives of a group of individuals are used to project how their pensions will evolve. There are a number of variants: dynamic and static, micro-simulations with behaviour, etc... Linked to these micro-simulation models are the projections based on individual life-cycle profiles generational accounting models, such as the one applied in Slovenia. It is often difficult to distinguish certain hybrid models which combine features of this model with those of aggregate accounting models. Micro-simulation models are used in France and Sweden.

3) General equilibrium models. The pension system is placed within an economic environment of general equilibrium with endogenous prices which generates explicit models of demographic and macro-economic evolution. The main drawbacks of these models are computational complexity, sensitivity to hypotheses and a clear shift away from the reality of the pension system, which means that they are rarely applied by official organisations. Holland is the exception where this type of model is applied.

4) Indirect models. These are based mainly on the internal rate of return or the transfer component and are usually applied to study intergenerational and intragenerational fairness.

The aim of this paper is to show the advisability of making it compulsory to draw up an actuarial balance in pay-as-you-go pension systems so as to improve their transparency, credibility and solvency. This is in line with the trend seen in some developed countries of trying to introduce actuarial analysis methodology into the field of public pay-as-you-go pension system management. The paper also aims to shed some light on the two main methods used by government Social Security departments to draw up the actuarial balance, focusing especially on their results, methodology and actuarial issues.

After this brief introduction, in Section 2 we define and explain the Swedish model, focusing especially on the balance sheet and the actuarial income statement, basic assumptions, actuarial aspects and the most recent results. In Section 3 we describe, define and present the basic aspects of the US actuarial balance, which we then compare with the Swedish and Japanese models. The paper ends with the main conclusions and four appendices, in which we show the main formulae applied for calculating the system's contribution asset and liabilities in the actuarial balance sheet in Sweden, and a simplified expression of the actuarial balance in the US and Japan.

## 2.-THE SWEDISH MODEL

The actuarial balance sheet for the PAYG pension system as compiled in Sweden does not fit into any of the methods briefly described above. It can be described as a financial statement listing the pension system's obligations towards contributors and pensioners at a particular date, with the amounts of the various assets (financial and through contributions) which back up these commitments.

As Boado-Penas et al. (2008) have pointed out, the main aim of the actuarial balance sheet is to give a true and fair view of the pension system's assets and liabilities at the beginning and end of the fiscal year and, by comparing these figures, to determine the change in net worth. It also contributes to management and disclosure of information as it is useful not only for the authority governing the system but also for contributors and pensioners in general, and also for whichever body guarantees payment, i.e. the State along with the contributors it represents.

For Settergren (2008), Swedish reporting on financial status bears greater resemblance to the standard income statement and balance sheet of an insurance company. Despite the fact that it reports on a pay-as-you-go plan, it also includes a solvency ratio, which is the ratio of assets to the pension liability. The income statement gives a full explanation of the reasons for changes in the system's solvency.

The main entries on the balance sheet are basically those shown in Table 1:

**Table 1: Main Entries on the Actuarial Balance Sheet of a Pay-As-You-Go System**

| ASSETS | LIABILITIES |
|---|---|
| Financial Assets | Liability to Pensioners |
| Contribution Asset | Liability to Contributors |
| Accumulated Deficit | Accumulated Surplus |
| Actuarial losses for the period | Actuarial profits for the period |
| **Total Assets** | **Total Liabilities** |

In general terms it can be said that a PAYG pension system is reasonably *solvent* as long as: (Financial + Contribution Asset) $\geq$ (Liability to Pensioners + Liability to Contributors). At the date of the balance sheet, therefore, the participants should have a realistic expectation of receiving the benefits that have been promised, without the system's sponsor (the State) having to make periodic additional contributions or the automatic balance mechanism (ABM) being triggered[5]. Solvency is clearly never completely assured in the long term as neither assets nor liabilities are known in their entirety.

The novel entry on the PAYG balance sheet is the one called the "contribution asset". This is derived from linking the pension system's assets and liabilities and is the result of a formula that shows the size of both the assets and the liabilities when the pension system is actuarially balanced and financed by pure pay-as-you-go, in a simplified scenario. It can be interpreted intuitively as *the maximum level of liabilities* that could be financed by *the existing contribution rate*, without periodic supplements from the sponsor, if the conditions prevailing at the time of valuation remain constant - in other words, Auerbach and Lee (2009), near or similar to the steady state[6].

Both the assets and the liabilities are valued on the basis of verifiable cross-section facts, i.e. no projections are made. For example, current longevity is used even though it is expected to increase. If and when that expectation materialises in new mortality tables, this will be incorporated into the

[5] According to Vidal-Meliá et al (2009b), the ABM is a set of predetermined measures established by law to be applied immediately as required according to the solvency indicator.

[6] If a system is in steady state, then the recently observed behaviour of the system will continue into the future. In the actuarial report on the Canada Pension Plan, see OSFIC (2008), the term "steady state" is used applied to the contribution rate but the meaning is not the same as usual. The steady-state contribution rate is defined as the lowest level contribution rate applicable after the end of the review period, which results in the asset/expenditure ratio being the same in the 10th and 60th year following the end of that review period.

information on the balance sheet on a year-to-year basis[7]. Because of this assumption, the calculation of the contribution asset does not take into account that contributions will grow in line with real salaries due to expected economic growth. This should not be interpreted as a belief that all the basic parameters determining the items on the balance sheet will remain constant in time, but as a result of the policy of using cross-section data. Changes are not included until they happen and can be verified. The Swedish National Social Insurance Board, Försäkringskassan (2002), argues that another advantage of using cross-section data is that it avoids the manipulations and biases that could affect any projections.

As already mentioned, producing an actuarial balance sheet is a practice that has been carried out in Sweden since 2001. The main data for the period 2004-2008 are shown in Table 2 below. The retirement contingency of the Swedish pension system is mixed, with 86.49% of the contributions being allocated to the PAYG system - NDC type - and the other 13.51% to the defined contribution capitalisation system. The balance sheet refers only to the PAYG part, notional type (Inkomstpension), and to the commitments deriving from the old pension system (ATP).

The 'financial asset' is the value of the financial assets owned by the Swedish pension system at the date of the balance sheet. It is valued according to internationally accepted principles, i.e. based on the financial prices of the securities held. It is of enormous size considering this is a PAYG system, amounting to 22.4% of GDP in 2008.

**Table 2: Balance Sheet of the Swedish Pension System at Dec. 31 of each year (ATP and Inkomstpension) for the period 2004-2008, in millions of SEK.**

| Year | 2008 | 2007 | 2006 | 2005 | 2004 |
|---|---|---|---|---|---|
| **ASSETS** | | | | | |
| **Financial Asset (F)** | 707,087 | 898,472 | 857,937 | 769,190 | 646,200 |
| **Contribution Asset (CA)** | 6,477,351 | 6,115,970 | 5,944,638 | 5,720,678 | 5,606,592 |
| **Actuarial losses (Table 3)** | 261,327 | 81,607 | ----- | ----- | 49,029 |
| **Total** | **7,445,765** | **7,096,049** | **6,802,575** | **6,489,868** | **6,301,821** |
| **LIABILITIES** | | | | | |
| **Liabilities to Contributors (AD)** | 5,156,684 | 4,909,569 | 4,750,749 | 4,612,959 | 4,486,030 |
| **Liabilities to Pensioners (DD)** | 2,271,123 | 2,086,915 | 1,952,261 | 1,848,517 | 1,757,979 |
| **Opening accumulated surplus** | 17,958 | 99,565 | 28,392 | 8,783 | 57,812 |
| **Actuarial profits** | ----- | ----- | 71,173 | 19,609 | ----- |
| **Total** | **7,445,765** | **7,096,049** | **6,802,575** | **6,489,868** | **6,301,821** |
| **GDP (in millions of SEK)** | | | | | |
| **GDP[8]** | **3,157,832** | **3,063,873** | **2,900,790** | **2,735,218** | **2,624,964** |
| **FUNDING AND SOLVENCY INDICATORS** | | | | | |
| **Solvency ratio** | **0.9672** | **1.0026** | **1.0149** | **1.0044** | **1.0014** |
| **Degree of funding %** | **9.52** | **12.84** | **12.80** | **11.90** | **10.35** |
| **(Liabilities to Contributors/Liabilities)%** | **69.4** | **70.2** | **70.9** | **71.4** | **71.8** |
| **Source: Försäkringskassan (2009), (2008), (2007), (2006), (2005), (2004), (2003), (2002) and own** | | | | | |

The value of the contribution asset is the product of the turnover duration (TD) and the value of the contributions made in that period. The TD is the time that is expected to pass from when a monetary unit enters the system as a contribution until it leaves in the form of a pension, assuming economic, demographic and legal conditions to be constant. It is also the sum of the weighted pay-in and pay-out durations[9] of one monetary unit in the system for the year's contributions, and is based on population data obtained from a cross-section, not a projection. In Sweden, to limit

[7] This is consistent with actuarial valuation basics. An actuarial valuation of a retirement plan (system) is an estimate of the plan's financial position at a specific point in time, and actuarial assumptions are primarily based on past experience or standard tables.
[8] This figure is introduced to give the reader some idea of the size of the pension system in relation to the size of the Swedish economy.
[9] See Appendix I.

fluctuations in the pension system's annual result, the contribution flow used in the calculation of the contribution asset is smoothed. If the population declines (increases), there is a risk that the accounts will (slightly) overstate (understate) the system’s assets in relation to its liabilities, since in such a case the turnover duration is (slightly) overestimated (underestimated). However, as the balance sheet is compiled every year according to verifiable events and transactions, it tends to provide a true and fair view. The stationary demographic and economic state is for sure not ex-post facto true, but because successive changes are included as they are registered in successive balance sheets, the solvency indicator remains reliable[10].

The “liability to contributors” is the notional capital accumulated in the contributors' accounts and that deriving from commitments to contributors under the old system (see Appendix II), and the “liability to pensioners” is the present value of the amount of all pensions in payment to current pensioners, taking into account current life expectancy and the real technical interest rate to be applied[11] (1.6%) when the amount of the initial pension was calculated and which is subsequently deducted from the yearly indexation of the pension. The liability to contributors amounts to 69.4% of total liabilities.

As can be seen from the balance sheet (Table 2), the Swedish system's degree of capitalisation (funding) (F/(AD+DD)) is remarkable, amounting to 9.52% of the liabilities in 2008. This enables any possible yearly imbalances between the system's income and expenditure to be dealt with by selling financial assets, which would make the need for outside financing unlikely, whether from the State or the financial market.

**Table 3: (Inkomstpension + ATP). Actuarial Income Statement at Dec. 31 2008.**
**Millions of SEK.**

| **FUND ASSETS (changes)** | | **-191,385** | **431,323** | **PENSION LIABILITIES (changes)** | |
|---|---|---|---|---|---|
| Contributions | 203,140 | | | 217,680 | New pension credit and ATP points[12] |
| Pension disbursements | -199,206 | | | 385,378 | Indexation[13] |
| Return on funded capital | -193,931 | | | 27,044 | Longevity[14] |
| Deduction for administrative costs | -1,388 | | | 1,345 | Inheritance gains (distributed - arising)[15] |
| **CONTRIBUTION ASSET (changes)** | | **361,381** | | -942 | Deduction for administrative costs |
| Contribution revenue | 394,833 | | | -199,182 | Pension disbursements |
| Turnover duration (TD) | -33,452 | | | | |
| **ACTUARIAL LOSSES** | | **261,327** | **0.0** | **ACTUARIAL PROFITS** | |
| **Total** | | **431,323** | **431,323** | **Total** | |

**Source: Own based on Försäkringskassan (2009).**

The accumulated surplus is the ‘accumulated profit’ or net worth of the pension system, which is owned by the system’s sponsor, in this case the State. As can be seen in Table 3, the system’s annual profit or loss is the difference between the increase in assets and the increase in liabilities during the period. The loss is also identical to the increase in the accumulated deficit or the

[10] See the papers by Auerbach and Lee (2008 and 2009)

[11] The classic actuarial discount factor is not used. The Försäkringskassan (2002) shows that a so-called “economic divisor” is used which takes into account the amount of pensions to be paid at each age for each individual, and which in the case of Sweden supplies a slightly different value to that of the classic actuarial discount factor.

[12] This does not coincide exactly with the contributions made due to the fact that the pension points will continue to be earned in the old (DB) system (ATP) until 2018. From then on the value of contributions will equal the value of pension credits, with only small differences due to administrative reasons.

[13] This is derived basically from changes in the average salary as measured by the income index used for indexing notional accounts and pensions (the latter with the 1.6% reduction).

[14] This is derived from changes in life expectancy included in the so-called “economic divisor”. See Appendix II

[15] The pension balances of deceased persons (inheritance gains arising) are distributed to survivors of the same age. This distribution is made as a percentage increase in pension balances by an inheritance gain factor. Due to lags in the recording and distribution of inheritance gains and the fact that, for persons over 60, statistically measured rather than actually incurred mortality is used, there are small discrepancies between inheritance gains arising and inheritance gains distributed.

reduction in the accumulated surplus, depending on the situation. The results are affected by economic and demographic factors. Usually, the main factor in the short term is growth in employment, whereas in the long term it is demographic factors. The principal cause for the year's losses is the decrease in the value of financial assets due to the current financial crisis. The shorter turnover duration has also affected the outcome. It seems clear that the income statement provides a full description of the reasons for changes in the system's solvency.

It is important not to confuse this profit or loss with the annual cash deficit or surplus. In Table 3, the cash deficit or surplus is the difference between the contributions received and the pensions paid. In 2008 the cash surplus amounted to (203,140-199,206) 3,934 million Swedish kronor, approximately 0.12% of GDP for that year. The system had "losses" in 2002, 2004, 2007 and 2008, and "profits" in 2003, 2005 and 2006. The initial figure for the accumulated surplus in 2001, the year of the first actuarial balance sheet, was obtained from the difference between all the assets and liabilities as a whole.

The main reason for valuing assets and liabilities without taking the future into consideration is that the system's financial solvency does not depend on the amount of assets and liabilities taken separately, but the ratio between them as measured by the solvency ratio.

The solvency ratio ($SR_t$) indicator used (Table 2) emerges from the actuarial balance sheet and, as mentioned before, does not rely on explicit projections of demographic and economic factors. It is expressed as[16]:

$$SR_t = \frac{\text{Assets}}{\text{Liabilities}} = \frac{\overbrace{F_t}^{\text{Financial Asset}} + \overbrace{AC_t}^{\text{Contribution Asset}}}{\underbrace{AD_t}_{\text{Liabilities to Contributors}} + \underbrace{DD_t}_{\text{Liabilities to Pensioners}}}$$

$$= \frac{F_t + (\overbrace{C_t}^{\text{Average contribution revenue}} \times \overbrace{TD_t}^{\text{Median of turnover duration}})}{(\underbrace{K_t}_{\text{Sum of the pension balances}} + \underbrace{F_t + ATP_t}_{\text{Liabilities to Contributors due to the pre-existing rules}}) + DD_t} \qquad \textbf{[1]}$$

The solvency ratio used in Sweden has a double purpose: to measure whether the system can fulfil its obligations to its contributors and to decide whether the automatic balance mechanism (ABM) should be applied.

The actuarial balance sheet of the Swedish pension system shows that the system is not fully solvent given that, at the date of the balance sheet, the pension liabilities can not reasonably be covered by the flow of income from contributions and the financial assets. Due to the heavy financial losses caused by the economic crisis of 2008, the system is not fully solvent. Judging from the balance sheet, contributors and pensioners do not have reasonable expectations that they will receive the pensions foreseen.

Because the solvency ratio is less than 1 (0.9672), the ABM will be activated in 2010. As a result, disbursements of the Inkomstpension and ATP during 2010 will be reduced by 3.28% in comparison to a situation without balancing, although the reduction will be partly offset by a higher guaranteed pension. The activation of the ABM will not be affected by any increase (decrease) in the value of the financial assets in 2009 because the relevant date for the valuation of the financial assets of the Inkomstpension (and for the balancing) is the last day of 2008. It seems clear that this activation will be the real test of the system's political solidity.

---

16 According to Auerbach and Lee (2009), the indicator is transparent because it is based on current cross- sectional data and is therefore less likely to be distorted by political pressures. See Appendices I and II for more details on calculating the solvency ratio.

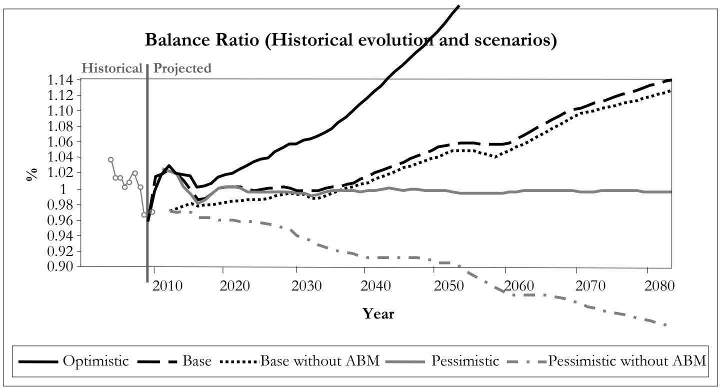


**Figure 1: Historical and Projected Balance Ratio for Sweden. Period 2008-2082.**

Projections of the system's possible future evolution are in fact made in the Swedish pension system's annual report. A projection is made of the balance sheet itself, of the amount in the reserve or "buffer fund" and the cash deficit or surplus, on the basis of three possible scenarios - normal (best estimate), pessimistic and optimistic - which provide valuable information[17]. However, this information is not used in the preparation of the actuarial balance sheet. It would be very difficult to justify a reduction in pension in real terms or an increase in the expected value of contributions made on the basis of a projection (or projected balance sheet) that may or may not be accurate.

As can be seen in Figure 1, in the base scenario the balance ratio for 2010, reflecting the balance at 31 December 2008, has dropped significantly, from a level just above (1.0026) in 2007 to 0.9672. This is largely, but not solely, due to heavy losses incurred by the buffer funds in 2008. In the projection, the balance ratio - partly due to the reduced indexation caused by the balance mechanism - will bounce back but stay close to 1 until the mid-2030s. After 2070 the balance ratio exceeds 1.1, a level which, as proposed by the government report "Distribution of Surpluses in the Inkomstpension System", means that there is a surplus distributable between contributors and pensioners. In the projections made in 2007, the balance ratio level had already reached 1.1 by 2037, indicating the sensitivity of the projections relative to the starting values. The main difference between the projections from 2007 and those from 2008 is the 20% smaller initial buffer fund. In the pessimistic scenario, the balance mechanism is more or less permanently triggered. With balancing, the system's liabilities accrue interest at the same rate as the growth in the system's assets. As a result, the balance ratio stabilises at around 1.0. In the projection at 31 December 2007, in the pessimistic scenario, the balance ratio fell below 1.0 in 2025.

## 3.- THE US MODEL IN COMPARISON TO THE JAPANESE MODEL

In this section we explore another example of actuarial balance, focusing on the US model and comparing it to the case of Japan.

The methodology used to compile the actuarial balance would best be described as a projection model of aggregate accounting spending on pensions, or an actuarial model. Basically it involves using the forecast demographic scenario to determine the future evolution of the number of contributors and pensioners according to the rules of the pension system. The macroeconomic scenario that determines the amounts of future contributions and pensions is exogenous.

The actuarial balance of the US Old-Age and Survivors Insurance (OASI) and Disability Insurance (DI) social security programs is aimed at measuring the system's financial solvency with a 75-year time horizon. It measures the difference in present value - discounted by the projected yield on trust fund assets - between spending on pensions and income from contributions, expressed as a percentage of the present value of the contribution bases for that time horizon, taking into account

[17] The results of the projections are also reported as calculations of net contributions and average pension levels for new expected pensioners.

that the level of financial reserves (trust fund) at the end of the time horizon reaches a magnitude of one-year's expenditure. The value summarises the system's financial deficit or surplus for the 75-year horizon and only for that horizon, and it therefore allows for a sharp jump[18] in the contribution rate or in the benefit level at the end of the 75-year period, and the winding-up of the trust fund on that date. If the balance is negative, the figure can be interpreted as the increase which would need to be applied to the contribution rate - immediately from that moment - in order to finance predicted benefits until the end of the 75-year period. The balance can also be expressed as the decrease in pensions, to be applied immediately, that would be needed for the contribution rate not to change within the next 75 years.

The report from which this actuarial balance is compiled is actually much more detailed. It contains a complete analysis of the assumptions used, the underlying methods and the long-term sensitivity of the main assumptions, and a stochastic actuarial balance is drawn up too.

In Japan, the actuarial balance or report is compiled at least every five years with a 95-year time horizon and, despite the fact it resembles the US version, it includes some important differences[19]. According to the AAD (2005a), Japan applies what is known as the "limited balance" or "closed period balancing" method[20]. Its public pension systems (Employees' Pension Insurance, National Pension and other public pension plans) have adopted the limited balance method when balancing expenditure with revenue. The limited balance method defines the balancing period, stipulated by law, as a period (95 years) in which most people who are already born at the valuation date will be dead, and during which expenditure needs to be balanced with revenue[21]. In the last fiscal year of the period of financial equilibrium (2100 in the case of the 2004 actuarial valuation), the reserve fund for payment must be equivalent to one year of disbursements.

The method for financial balancing is carried out by the so-called "fixed contribution programme method" and "modified indexation": contributors will receive benefits whose level is automatically adjusted according to the rate by which the number of active participants in the social security pension schemes decreases and the rate by which longevity at age 65 improves until financial equilibrium is attained under the fixed contribution programme. However, as benefit levels may be lowered too much if pensions are subject to modified indexation with the contribution programme, the pension scheme defines the lower limit of benefit levels and stipulates that the contribution-benefit regime of the scheme should be drastically reviewed when the benefit level threatens to fall below predefined levels by the next actuarial valuation.[22]

---

[18] In fact the 75-year time horizon moves ahead every year and the deficit will be noticed well before the end of the time horizon or the sharp jump materialises. If they do nothing with the programme despite recognising the deficit, the trust fund will be depleted well before the end of the 75-year time horizon, and a sharp rise in the contribution rate or a steep decline in benefit levels will take place on the fund depletion date.

[19] Adoption of the Swedish type actuarial balance also depends on the socio-economic conditions of the country concerned. For example, Japan is ageing much faster than Sweden and the Swedish automatic balance mechanism would probably be continuously activated if it were applied to Japanese social security pension schemes. Furthermore, the adjustment degree on each balancing might be too small to restore long-term financial equilibrium and it would take a long time for this equilibrium to be restored. Another problem with Japan is the benefit design. It is composed of a flat-rate part and an earnings-related part with a re-distributional function. It also provides disability and survivors' benefit. Under such a design it would be difficult or even impossible to define the turnover duration (TD). For these reasons, Sakamoto (2005), Japan did not adopt the Swedish actuarial balance sheet and automatic balance mechanism. As a result, the Japanese automatic balance mechanism was constructed in a different way, which was, by coincidence, quite similar to the German method based on the sustainability factor, though the approach was quite different. The pension reforms of Germany and Japan also coincidentally took place in the same year, 2004.

[20] The closed period balancing method means that the period of financial equilibrium is finite, whereas the whole future balancing method considers that the period of financial equilibrium is for a perpetual time horizon, and was applied in Japan for the actuarial valuation in 1999.

[21] See Formula 21 in Appendix IV.

[22] A mechanism for automatically adjusting the benefit level according to future changes in social and economic conditions - so-called "modified indexation" - has been incorporated into the pension system. See Sakamoto (2005, 2008) and Vidal-Meliá et al. (2009a) for details. The replacement rate specifically defined for monitoring the benefit level is forecast to decrease from 59.3% in 2004 to 50.2% in 2023. In order to prevent the benefit level from becoming too low, the law stipulates the provision of a minimum benefit level: if the replacement rate threatens to fall below 50% before implementation of the next actuarial review, "modified indexation" is to cease to be applied, and the scheme is to be given a drastic review.

According to the US actuarial balance at 31-12-2008 under intermediate assumptions (so-called "best-estimate" assumptions [23], see Table 4), the system could regain financial solvency in 75 years if a 2.00-point increase in the contribution rate were to be implemented immediately, applied to taxable earnings. The actuarial balance (see Formula 19 in Appendix 3) is the difference between the summarised income rate and the summarised cost rate over a given valuation period. The adjusted summarised income rate (14.02%) equals the ratio of (a) the sum of the trust fund balance at the beginning of the period plus the present value of the total income from taxes during the period[24], to (b) the present value of the taxable payroll for the years in the period. The adjusted summarised cost rate (16.02%) is equal to the ratio of (a) the sum of the present value of the cost during the period plus the present value of the targeted ending trust fund level, to (b) the present value of the taxable payroll during the projection period.

**Table 4: Elements of the 75-Year Actuarial Balance. Under Intermediate Assumptions (2009-2083). Present value as of January 1 2009, in billions of dollars[25].**

| ITEM | | OASDI |
|---|---|---|
| 1 | **Payroll tax revenue** | 35,041 |
| 2 | **Taxation of benefits revenue** | 2,175 |
| 3=1+2 | **Tax income** | 37,216 |
| 4 | **Cost** | 44,894 |
| 5=-4+3 | **Initial deficit** | -7,678 |
| 6 | **Trust fund assets at start of period** | 2,419 |
| 7=5+6 | **Open group unfunded obligation[26]** | -5,259 |
| 8 | **Ending target trust fund[27].** | 402 |
| 9=7-8 | **Results for the period[28]** | -5,661 |
| 10 | **Taxable payrolls** | 282,781 |
| 11=(3+6)/(10)% | **Summarised income rate** | **14.02%** |
| 12=(4+8)/(10)% | **Summarised cost rate** | **16.02%** |
| 13=(9/10)% | **Deterministic actuarial balance** | **-2.00%** |
| Details in BOT (2009) | **Deterministic actuarial balance (TH ∞)** | -3.41% |
| | **Stochastic actuarial balance, 50th percentile (TH 75 years)** | -2.16% |

**Source: BOT (2009) and own.**
**Totals do not necessarily equal the sums of rounded components.**

Similarly, all expected payments could be made up to 2083 if an across-the-board cut of 13.3% were imposed on benefits[29] or an allocation of 5,3 trillion $US were made to the "trust fund". Naturally a combination of both these measures could be made instead. In terms of annual deficit or surplus, a cash deficit is forecast to appear in 2016 and the reserve fund is expected to be exhausted in 2037.

The result of the actuarial balance for a perpetual time horizon is -3.41%, and the unbacked liabilities are estimated at 15.1 trillion dollars. The actuarial balance estimated using stochastic methodology for a 75-year time horizon gives a result of -2.16% for the 50th percentile, unbacked liabilities total $5.7 trillion dollars, and the reserve fund is forecast to be exhausted in 2036, i.e.

[23] According to standard actuarial terminology, the "best-estimate" scenario is that obtained under "best-estimate assumptions". These assumptions reflect the best judgement of the experts as to future demographic and economic conditions that will probably affect the long-term financial solvency of the system.
[24] This represents 17.17 % of projected GDP in 2009.
[25] A billion dollars is equal to $1 * 10^9$ $, and a billion euros would be $1 * 10^{12}$ € . A trillion dollars is equal to $1 * 10^{12}$ $, and a trillion euros would be the much greater figure of $1 * 10^{18}$ €.
[26] Present value of the debt that would have to be incurred to fund the payments that have been promised, wiping out all the financial assets. This should not be confused with the implicit debt of the system at a particular date.
[27] The calculation of the actuarial balance includes the cost of accumulating a target trust fund balance equal to 100 percent of annual cost by the end of the period.
[28] This represents 40.18% of GDP in 2009 (14,088 billion dollars), or 0.72% of the present value of the GDPs for the period 2009-2083 (788.4 trillion dollars).
[29] This cannot be directly obtained from Table 4. See details of calculation in BOT (2009).

everything is similar to the result of the determinist actuarial balance in the case of intermediate assumptions. The confidence interval of 95% indicates that the value of the actuarial balance swings between (-3.98% and -0.56%), and this range is smaller than would be the case for the best and worst assumptions for the system (-5,32% and +1,18%).

Figure 2 shows in the second vertical axis the evolution of the actuarial balance over the last 27 years. Although the value of the balance for 2008 was the best for the last 15 years, comparisons between values are not fully homogeneous due to the fact that practically every year methodological improvements are incorporated which prevent direct comparison, although details of the changes and their year-on-year effects are supplied in the report from which the actuarial balance is drawn. Even so, the chart gives an effective summary of each year's expectations as to the evolution of the system's financial health over the following 75 years.

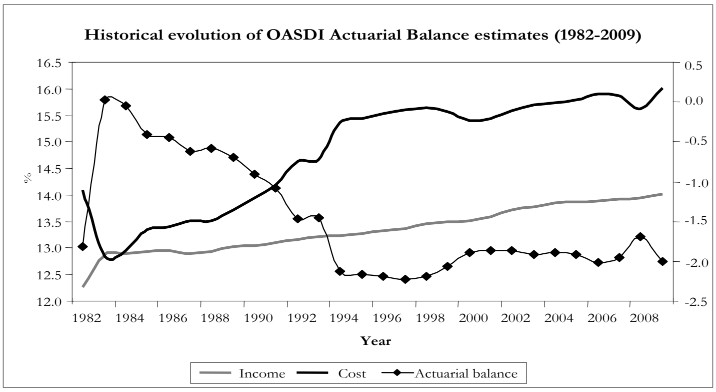


**Figure 2: Historical evolution of OASDI actuarial balance estimates (1982-2009). Intermediate Assumption.**

Table 5 shows the past and forecast evolution for the intermediate assumption of some of the key elements that have an effect on determining the actuarial balance[30].

The income rate is the ratio between the income from tax revenues on a liability basis (payroll tax contributions and income from the taxation of scheduled benefits) and the OASDI taxable payroll for the year. The cost rate for a year is the ratio between the cost of the programme and the taxable payroll for the year. The annual balance is the difference between the income rate and the cost rate in a given year. As can be seen in the table, the annual balance worsens considerably insofar as the ratio of contributors to beneficiaries decreases due to the retirement of the large baby-boom generation between about 2011 and 2030. After 2030, increases in life expectancy and the relatively low fertility rates since the baby boom cause the level of the accumulated financial reserves to fall noticeably until they become exhausted, as shown in the value of the "trust fund ratio", defined as the assets at the beginning of a year (including advance tax transfers if any) expressed as a percentage of the cost during the year. The trust fund ratio represents the proportion of a year's cost which could be paid with the funds available at the beginning of a year.

**Table 5: Past figures and forecast evolution (intermediate assumption) of some of the basic OASDI indicators.**

| Years | Income rate % | Cost rate % | Annual Balance% | Contributors /Beneficiaries | Trust Fund ratio (Years) |
|---|---|---|---|---|---|
| **1990** | 12.49 | 10.74 | 1.75 | 3.4 | 0.75 |
| **1995** | 12.59 | 11.67 | 0.92 | 3.3 | 1.28 |
| **2000** | 12.69 | 10.40 | 2.29 | 3.4 | 2.16 |
| **2005** | 12.71 | 11.16 | 1.55 | 3.3 | 3.18 |
| **2007** | 12.75 | 11.32 | 1.43 | 3.3 | 3.45 |

[30] Information very similar to that shown in Table 5 is also published in Japan but, as mentioned previously, it is envisaged that for a 95-year time horizon the value of the "trust fund ratio" will be at least 1. See AAD (2006).

| | | | | | |
|---|---|---|---|---|---|
| **2008** | 12.71 | 11.38 | 1.33 | 3.2 | 3.58 |
| **2010** | 12.87 | 12.50 | 0.37 | 3 | 3.60 |
| **2020** | 13.04 | 14.50 | -1.46 | 2.5 | 3.15 |
| **2030** | 13.20 | 16.76 | -3.56 | 2.2 | 1.53 |
| **2040** | 13.25 | 16.99 | -3.74 | 2.1 | --- |
| **2050** | 13.25 | 16.61 | -3.36 | 2.1 | --- |
| **2060** | 13.27 | 16.73 | -3.46 | 2.1 | --- |
| **2070** | 13.30 | 17.05 | -3.75 | 2 | --- |
| **2080** | 13.33 | 17.53 | -4.20 | 2 | --- |
| **Source: BOT (2009) and own.** | | | | | |

If no action were taken until the combined trust funds were exhausted in 2037, then the effects of changes would be more concentrated in fewer years and fewer cohorts. For example, payroll taxes could be raised to fully finance scheduled benefits in every year starting in 2037. In this case, the payroll tax would be increased to 16.26% at the point of trust fund exhaustion in 2037 and continue rising to 16.74% in 2083. Similarly, benefits could be reduced to the level that is payable with scheduled tax rates in each year beginning in 2037. Under this scenario, benefits would be reduced by 24% at the point of trust fund exhaustion in 2037, with reductions reaching 26% in 2083. Either of these examples would eliminate the shortfall for the 75-year period as a whole by specifically eliminating annual deficits after trust fund exhaustion. Consequently, ensuring the system's solvency beyond 2083 would probably require further changes beyond those expected to be needed for 2083. All this can be seen in Figure 3.

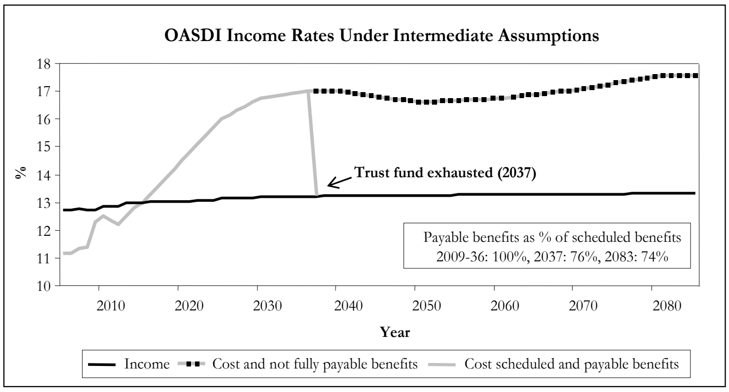


**Figure 3: OASDI Income and Cost Rates under Intermediate Assumptions as a Percentage of Taxable Payrolls.**

To summarise, the main differences between the Swedish, US and Japanese actuarial balances are the following:

1.-Projections of demographic, economic and financial variables are made for a 75-year period in the USA with no changes in legislation, while in Japan the period is 95 years and the changes in benefit level necessary to achieve financial equilibrium are incorporated into the projection since it is stipulated by law. In Sweden a valuation system based on verifiable facts at the effective date of the balance sheet is used.

2.-In Sweden the contribution asset is quantified in a stationary state, while in the USA and Japan contributions are estimated for the next 75 years and 95 years respectively.

3.-The actuarial balance in the USA and Japan is dependent on the market interest rate, but the Swedish balance sheet, being independent, is not.

4.-The Swedish balance sheet follows the traditional structure of the accounting balance sheet deriving from principles of double-entry bookkeeping and has a very strong actuarial profile as it includes its commitments to both pensioners and contributors as liabilities, while the US (and the

Japanese) balance has a more financial profile as its commitments to contributors are not quantified until those contributors become pensioners.

5.-Every year the information deriving from the Swedish balance sheet has an effect on the indexation of the contributions registered in the notional accounts and on the rate of variation of pensions in payment, while the US actuarial balance has no immediate effect but serves as an element to provoke thought and analysis for possible legislative reform of the pension system. In this respect the Japanese model resembles the Swedish rather than the US model as it introduces mechanisms for adjusting spending in line with income over the entire period considered (95 years). Therefore the Japanese actuarial report, despite the fact that it uses projections that may or may not be true, does have an immediate effect on present and future pensioners in that whether their indexation should be modified or not depends on the result of the actuarial valuation.

6.-The solvency indicators for the system emerging from both balance sheets can be complementary in some cases, although they are conceived and composed in clearly different ways. The US solvency indicator is no more than the sum in present value of the cash deficits or surpluses for the time period considered, to which are added the value of the financial assets at the beginning of the valuation period. The Swedish indicator concentrates more on evaluating the commitments it has with both pensioners and contributors. In the Japanese model, the indicator is the projected sum of the deficits, which by definition is 0 if reality corresponds to the assumptions, as it incorporates measures for balancing the system in the horizon considered.

## 4.-CONCLUDING COMMENTS

There are a number of useful instruments deriving from actuarial analysis methodology that can be applied to the public management of PAYG systems to improve their transparency, credibility and solvency, and the actuarial balance is undoubtedly one of them.

As regards transparency, the really important thing about the actuarial balance is what it represents (or should represent) for good pension system management: the fact that it has to be compiled every year, or at least every five years as in Japan, exceeding the traditional planning horizon of the politicians. Compiling actuarial balances - which in the cases described are also audited and have the support of experts on the subject – should "oblige" politicians to be much more careful about what they say and minimise the use of populism in pensions; unfortunately, the reality is much more complex[31]. In addition to this, it gives contributors and pensioners a reliable idea of how far promises made to them regarding the payment of their pensions are kept, and, undoubtedly, it makes them more involved in the way their payment evolves as they have greater knowledge of both the system and their individual rights and obligations.

On the subject of solvency, the obligation to compile an actuarial balance every year, or at least periodically as in Japan and Canada, makes people more interested in how it develops, and this may make it easier to introduce automatic balance mechanisms (ABMs) into the system to guide it firmly onto the road to long-term financial solvency. If this were not the case, it would be practically impossible to introduce them.

It seems more appropriate for the Swedish actuarial balance sheet to be applied to the NDC system, especially if measures that immediately affect current pensioners and contributors can be derived from the solvency indicator, which would be more difficult to justify if they were based on projections that need explicit assumptions of future developments that are easy criticise. There is

[31] For example, in Japan politicians are still trying to make pension matters a political football. The opposition parties insist that the 2004 reform is not a drastic reform as it left the 'anomaly' between employees and the self-employed. However, their specific proposals are not clear, especially on how to attach the income to the self-employed in an equitable way. Nevertheless, it appeals to some voters. In Sweden as well, the government, very concerned (in electoral terms) about the activation of the balance, is about to propose an amendment to parliament in order to change the valuation principles for the financial assets of the buffer fund: the 3-year average size of the reserve fund will probably be used to calculate the balance ratio. In Germany, modifying the indexation by the 'Riester step' was postponed to delay the reduction of benefits. They are all conscious of forthcoming elections.

nothing to prevent this actuarial balance sheet being applied to many DB PAYG systems[32], mainly if there is a clear separation between the retirement contingency and the others.

The US actuarial balance has a different mission from the Swedish one. Its aim is not to provide for automatic piloting but rather to provide information to the "interested public" and legislators, information on which the latter may or may not act. Another difference is the information provided by the two types of analysis. The more traditional double-entry bookkeeping method of the Swedish system provides information on changes in the pension plan's financial position - the net income of the accounting period - incurred during the accounting period, and in its income statement it quantifies the sources of that change. The future, any time after the last day of the accounting period, does not affect the analysis. The traditional actuarial projection of the US system is largely concerned with the future. The result of the US analysis is condensed into a single number - the actuarial balance - while the sources of the change in the actuarial balance are not as easily quantified as in the Swedish case. Nevertheless, such quantification of the sources of change is indeed provided in the US case, but does not follow from an accounting identity.

The Japanese actuarial report model includes elements that can also be found in the two models described earlier: it highlights future challenges to the system - mainly ageing and the forecast increase in longevity - through projections, but it incorporates automatic measures (so-called "modified indexation"' under the fixed contribution programmes) to deal with the projected future imbalance between the costs and income of the system. These measures can be adapted every five years in line with the projections contained in the new actuarial report.

Last but not least, it would be interesting and potentially productive politically to get official yearly information on these two types of actuarial balances for all public pension systems. In this respect ISSA, the World Bank, the OECD and other international organisations (for example, the Pension Benefits and Social Security (PBSS) section of the International Actuarial Association) could be supportive in developing and enforcing international accounting and actuarial valuation standards for pay-as-you-go pension systems.

## 5.-REFERENCES


1. Actuarial Affairs Division, Pension Bureau AAD (2005a). "Outline of 2004 actuarial valuation on Employees' Pension Insurance and National Pension in Japan" *Ministry of Health, Labour and Welfare.*

2. Actuarial Affairs Division, Pension Bureau AAD (2005b). "Actuarial Affairs of the Pension System" Chapter 8 *Ministry of Health, Labour and Welfare.*

3. Actuarial Affairs Division, Pension Bureau AAD (2006). "Review of the 2004 Actuarial Valuation of the Public Pension Plans (Summary). Pension Insurance and National Pension in Japan" *Ministry of Health, Labour and Welfare.*

4. Auerbach, A. J. and R. D. Lee (2009). "Welfare and Generational Equity In Sustainable Unfunded Pension Systems" *National Bureau of Economic Research*, WP-14682.

5. Auerbach, A. J. and R. D. Lee (2008). "Notional Defined Contribution Pension Systems in a Stochastic Context: Design and Stability," in J. Brown, J. Liebman, and D. Wise, eds., Social Security Policy in a Changing Environment. University of Chicago Press: Chicago, forthcoming.

6. Boado-Penas, C. (2008). "Instruments for Improving the Equity, Transparency and Sustainability of Pay-As-You-Go Pension Systems". European Doctoral Thesis. University of Valencia, Spain.

7. Boado-Penas, C, S. Valdés-Prieto and C. Vidal-Meliá (2008). "An Actuarial Balance Sheet for Pay-As-You-Go Finance: Solvency Indicators for Spain and Sweden". *Fiscal Studies*, 29, 89-134.

8. Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds (BOT) (2009). *2008 Annual Report*. Washington, D.C.: Government Printing Office.


[32] Boado-Penas et al. (2008) apply it to the Spanish contributory pension system and uncover its problems of structural actuarial disequilibrium.

9. Bovenberg, L.A. (2003). "Financing Retirement in the European Union". *International Tax and Public Finance*, 10, 713–734.

10. Lefebvre, C. (2007). "Projections à long terme des systèmes de retraite: quelques expériences étrangères". Rapport au Conseil d'orientation des retraites.

11. Office of the Superintendent of Financial Institutions Canada (OSFIC) (2008). "Actuarial Report (23rd) on the CANADA PENSION PLAN". Office of the Chief Actuary. http://www.osfi-bsif.gc.ca

12. Office of the Superintendent of Financial Institutions Canada (OSFIC) (2007). "Optimal Funding of the Canada Pension Plan". Actuarial Study N. 6 April, Office of the Chief Actuary. http://www.osfi-bsif.gc.ca

13. Sakamoto, J. (2005). "Japan's Pension Reform". Social Protection Discussion Paper 0541. The World Bank.

14. Sakamoto, J. (2008). "Roles of the Social Security Pension Schemes and the Minimum Benefit Level under the Automatic Balancing Mechanism". NRI Papers No. 125 presented at the Joint Colloquium of the IACA, PBSS and IAAHS Sections of the International Actuarial Association, Boston, USA, 4-7 May 2008.

15. Settergren, O (2008). "Optimal financing of social security pension schemes and its design. Defining meaningful information to assess the financial status of social security pension schemes". Presented al the Technical Seminar for Social Security Actuaries and Statisticians, Limassol, Cyprus, 30-31 October 2008.

16. Settergren, O (2003). "Financial and Inter-Generational Balance? An Introduction to How the Swedish Pension System Manages Conflicting Ambitions." *Scandinavian Insurance Quarterly* 2, 99–114.

17. Settergren, O (2001). "The Automatic Balance Mechanism of the Swedish Pension System – a non-technical introduction". *Wirtschaftspolitische Blätter* 4/2001, 339-349.

18. Settergren, O. and B.D. Mikula (2005). "The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest" *The Journal of Pensions Economics and Finance*, 4 (2), 115–138

19. Takayama, N. (2005). "The balance sheet of social security pensions in Japan", Proceedings No. 6, "The Balance Sheet of Social Security Pensions", *Institute of Economic Research*, Hitotsubashi University, February, 2005.

20. The Economic Policy Committee TEPC (2007). "Pensions Schemes and Projection Models in EU-25 Member State". European Economy, Occasional Paper, 35 November.

21. The Swedish Pension System Annual Report 2001. (2002). Ed. Ole Settergren, National Social Insurance Board (Försäkringskassan), Stockholm

22. The Swedish Pension System Annual Report 2002. (2003). Ed. Ole Settergren, National Social Insurance Board (Försäkringskassan), Stockholm.

23. The Swedish Pension System Annual Report 2003. (2004). Ed. Ole Settergren, National Social Insurance Board (Försäkringskassan), Stockholm.

24. The Swedish Pension System Annual Report 2004. (2005). Ed. Ole Settergren, National Social Insurance Board (Försäkringskassan), Stockholm.

25. The Swedish Pension System Annual Report 2005. (2006). Ed. Ole Settergren, National Social Insurance Board (Försäkringskassan), Stockholm.

26. The Swedish Pension System Annual Report 2006. (2007). Ed. Ole Settergren, National Social Insurance Board (Försäkringskassan), Stockholm.

27. The Swedish Pension System. Orange Annual Report 2007. (2008). Ed. Ole Settergren, Swedish Social Insurance Agency (Försäkringskassan), Stockholm.

28. The Swedish Pension System. Orange Annual Report 2008. (2009). Ed. Anika Sunden, Swedish Social Insurance Agency (Försäkringskassan), Stockholm

29. Vidal-Meliá, C., M.C. Boado-Penas and O. Settergren (2009a). "Automatic Balance Mechanisms in Pay-As-You-Go Pension Systems". *The Geneva Papers on Risk and Insurance: Issues and Practice.* 33 (4), 287-317.

30. Vidal-Meliá, C., M.C. Boado-Penas and O. Settergren (2009b). "Instruments for improving the equity, transparency and solvency of pay-as-you-go pension systems: NDCs, ABs and ABMs." Chapter 18 in M. Micocci, G. N. Gregoriou and G. B. Masala, eds., *Pension Fund Risk Management. Financial and Actuarial Modelling*. Chapman and Hall. Forthcoming

**APPENDIX I: CONTRIBUTION ASSET FOR THE CASE OF SWEDEN[33].**

$$CA_t = \overline{C}_t \times \overline{TD}_t \qquad \textbf{[2]}$$

where:

t: Calendar year if the variable refers to flows, end of calendar year if the variable refers to stocks; CA: Contribution asset; C: Contribution revenue, year t; TD: "turnover duration"; $\overline{C}$: Average contribution revenue of the last three years; and $\overline{TD}$: Median of TD to the PAYG pension system for the last three years.

$\overline{C}$ and $\overline{TD}$ are:

$$\overline{C}_t = \frac{C_t + C_{t-1} + C_{t-2}}{3} \times \left( \frac{C_t}{C_{t-3}} \times \frac{RPI_{t-3}}{RPI_t} \right)^{1/3} \times \left( \frac{RPI_t}{RPI_{t-1}} \right) \qquad \textbf{[3]}$$

$$\overline{TD}_t = \text{median}\left[ TD_{t-1}, TD_{t-2}, TD_{t-3} \right] \qquad \textbf{[4]}$$

where,

RPI: Retail price index for June, year t.

The TD can be disregarded in two sub-periods: "pay-in duration", $pt_{c,t}$, and "pay-out duration", $pt_{r,t}$,. The sum of both concepts would form the average turnover duration of one monetary unit (m.u) and would also be equivalent to the difference between the average weighted age for the pensioners (weighted by pension size that takes into account the age–benefits profile) and contributors (weighted by contribution sizes that take into account the age–earnings profile) at the end of year t[34] ($A_{r,t} - A_{c,t}$):

$$TD_t = pt_{c,t} + pt_{r,t} = A_{r,t} - A_{c,t} \qquad \textbf{[5]}$$

The expression for $pt_c$ is:

$$pt_c = \frac{\sum_{i=16}^{\overline{R}_t - 1} \overline{E}i_t \times Li_t \times (\overline{R}_t - i - 0{,}5)}{\sum_{i=16}^{\overline{R}_t - 1} \overline{E}i_t \times Li_t} \qquad \textbf{[6]}$$

where

[33] See Försäkringskassan (2009).
[34] See Settegren (2003) and Settergren & Mikula (2005).

$$\overline{R}_t = \frac{\sum_{i=61}^{R_t^*} P_{i,t}^* \times G_{i,t} \times i}{\sum_{i=61}^{R_t^*} P_{i,t}^* \times G_{i,t}} \quad \textbf{[7]}$$

$\overline{R}$ : Average retirement age weighted by the amount of the pension of those that reach retirement in this period, rounded off to nearest whole number.

$$\overline{E}_{i,t} = \frac{\frac{E_{i,t}}{N_{i,t}} + \frac{E_{i+1,t}}{N_{i+1,t}}}{2}, \quad i = 16, 17, \ldots, \overline{R}_{t-2}, \; \overline{E}_{\overline{R}(t)-1,t} = \frac{E_{\overline{R}(t)-1,t}}{N_{\overline{R}(t)-1,t}} \quad \textbf{[8]}$$

$$L_{i,t} = L_{i-1,t} \times h_{i,t}, \quad i = 17, 18, \ldots, \overline{R}_{t-1}, \text{ with } L_{16,t} = 1 \quad \textbf{[9]}$$

$$h_{i,t} = \frac{N_{i,t}}{N_{i-1,t-1}} \quad i = 17, 18, \ldots, \overline{R}_{t-1} \quad \textbf{[10]}$$

where,

i: Age at year-end; $R_t^*$ : Oldest age group for which pensions have been granted in year t; $P_{i,t}^*$ : Total of pensions granted monthly in year t to persons in age group i; $E_{i,t}$ : Sum of 16% of pension qualifying-income year t age group i; $N_{i,t}$ :Number of individuals in age group i who at any time during pay-in year t have been credited with pension-qualifying income and have not been registered as deceased; $L_{i,t}$ : Proportion of persons in age group i in year t; $h_{i,t}$ : Change in proportion of persons in age group i in year t; and $G_{i,t}$ : Annual demographic divisor in year t for pensioner group aged i. In calculating the initial pension, the notional capital accumulated in the contributors' accounts is divided by the "demographic divisor", whose expression is as follows:

$$G_{i,t} = \frac{1}{12 L_i} \sum_{k=i}^{R_t} \sum_{X=0}^{11} \left[ L_k + \frac{L_{k+1} - L_k}{12} X \right] (1{,}016)^{-(k-i)} (1{,}016)^{-X/12} \text{ for } i = 61, 62, \ldots, R_t \quad \textbf{[11]}$$

where $L_i$ : Number of survivors in age group i, according to the life-span statistics for Sweden with real data of the last five years; $\frac{L_{k+1} - L_k}{12} X$ : takes into account the monthly disbursements; $i$ : Retirement age; $k - i$ : Number of years of retirement; and $X$ : Months (0, 1, 2, 3, …, 11)

It can be proven, Boado-Penas (2008), that this "demographic divisor" is equivalent to the present value of a life annuity due of 1, constant, in instalments, with a technical interest rate of 1.6%.

The expression for $pt_r$ is:

$$pt_{r,t} = \frac{\sum_{i=\overline{R}_t}^{R_t} 1{,}016^{-(i-\overline{R}_t)+0{,}5)} \times L_{i,t}^* \times (i - \overline{R}_t + 0{,}5)}{\sum_{i=\overline{R}_t}^{R_t} 1{,}016^{-(i-\overline{R}_t)+0{,}5)} \times L_{i,t}^*} \quad \textbf{[12]}$$

$$L_{i,t}^* = L_{i-1,t}^* \times he_{i,t}, \; L_{60,t}^* = 1 \quad \textbf{[13]}$$

$$he_{i,t} = \frac{P_{i,t}}{P_{i,t} + Pd_{i,t} + 2 \times Pd^*_{i,t}} \quad \text{for } i = 61, 62, \ldots, R_t \qquad [14]$$

with: $R_t$ : Oldest age group receiving a pension in year t, $P_{i,t}$ : Total pension disbursements of year t to age group i; $Pd_{i,t}$ : Total of the last monthly pension disbursements to persons in age group i who received pensions in December of year t-1 but not in December of year t; $Pd^*_{i,t}$ : Total of the last monthly pension disbursements to persons in age group i who were granted pensions in December of year t-1 and did not receive a pension payment in December of year t; $L^*_{i,t}$ : Proportion of remaining disbursements to age group i in year t; and $he_{i,t}$ : Change in pension disbursements due to deaths in year t, age group i.

**APPENDIX II: PENSION LIABILITIES FOR THE CASE OF SWEDEN**

$$D_t = AD_t + DD_t \qquad [15]$$

$$AD_t = K_t + E_t + ATP_t \qquad [16]$$

$$DD_t = \sum_{i=61}^{R_t} P_{i,t} \times 12 \times \left( \frac{Ge_{i,(t)} + Ge_{i,(t-1)} + Ge_{i,(t-2)}}{3} \right) \qquad [17]$$

$$Ge_{i,(t)} = \frac{\sum_{j=i}^{R_t} \frac{1}{2} \times (L^*_{j,t} + L^*_{j+1,t})\, 1{,}016^{-(i-j-1)}}{L^*_{i,t}} \quad \text{for } i = 61, 62, \ldots, R_t \qquad [18]$$

where

$AD_t$ : Pension liability in year t with regard to pension commitment for which disbursement has not yet commenced (pension liability to the economically active).

$DD_t$ : Pension liability in year t with regard to pensions being disbursed to retired persons in the PAYG system.

$K_t$ : The sum of the pension balances of all insured persons in year t.

$E_t$ : Estimated pension credit for the Inkomstpension earned in year t, according to Swedish law.

$ATP_t$ : Estimated value of the ATP in year t for persons who have not yet begun to receive this pension. Persons born before 1938 who have not earned either an Inkomstpension or a premium pension. Instead they receive the ATP, which is calculated by pre-existing rules. The ATP liability to the economically active will gradually diminish and will in principle be gone entirely by 2018.

$Ge_{i,(t)}$ : Economic annuity divisor for age group i in year t. In Sweden, for calculating the debt with current pensioners, the initial pension of each cohort is multiplied by the "economic divisor" for that cohort, which corresponds to an actuarial income weighted by the number of pensioners with their respective pensions. This should not be confused with the "demographic divisor" described earlier.

**APPENDIX III: THE ACTUARIAL BALANCE FOR THE CASE OF THE USA**

In simplified form, the actuarial balance (AB) can be expressed as:

$$AB=\frac{\left[\overbrace{TF_0}^{\text{Trust fund}}+\overbrace{y_0\sum_{t=0}^{74}\theta_t N_t\prod_{h=1}^{t}\frac{(1+g_h)}{(1+r_h)}}^{\text{Present value of contributions}}\right]-\left[\overbrace{B_0\sum_{t=0}^{74}R_t\prod_{h=1}^{t}\frac{(1+\lambda_h)}{(1+r_h)}}^{\text{Present value of benefits}}+\overbrace{\prod_{h=1}^{74}\frac{(TF_{74})}{(1+r_h)}}^{\text{Trust fund}}\right]}{\underbrace{y_0\sum_{t=0}^{74}N_t\prod_{h=1}^{t}\frac{(1+g_h)}{(1+r_h)}}_{\text{Present value of payrolls}}} \qquad \textbf{[19]}$$

$$=\underbrace{\left[\frac{TF_0+y_0\sum_{t=0}^{74}\theta_t N_t\prod_{h=1}^{t}\frac{(1+g_h)}{(1+r_h)}}{y_0\sum_{t=0}^{74}N_t\prod_{h=1}^{t}\frac{(1+g_h)}{(1+r_h)}}\right]}_{\text{Summarised income rate}}-\underbrace{\left[\frac{B_0\sum_{t=0}^{74}R_t\prod_{h=1}^{t}\frac{(1+\lambda_h)}{(1+r_h)}+\prod_{h=1}^{74}\frac{(TF_{74})}{(1+r_h)}}{y_0\sum_{t=0}^{74}N_t\prod_{h=1}^{t}\frac{(1+g_h)}{(1+r_h)}}\right]}_{\text{Summarised cost rate}}\approx 0$$

which in a situation of financial equilibrium for the valuation period should give a zero value, where:

$TF_0$ : Value of assets at the beginning of the valuation period; $\theta_t$ : Payroll tax (contribution) rate in year t; $y_0$ : Average contribution base in year 0; $N_t$ : Number of contributors in year t; g: Annual real wage growth rate; r: Projected yield on trust fund assets; $B_0$ : Average pension (benefit) in year 0; $R_t$ : Number of pensioners in year t; and $\lambda$: Annual real benefit growth rate.

With the data from the numerator in Formula [19], a solvency indicator similar to the Swedish one can be constructed immediately, which in a situation of solvency should give a unitary value.

$$AB=\frac{\left[\overbrace{TF_0+y_0\sum_{t=0}^{74}\theta_t N_t\prod_{h=1}^{t}\frac{(1+g_h)}{(1+r_h)}}^{\text{Present value of incomes}\approx\text{Assets for the valuation period}}\right]}{\left[\underbrace{B_0\sum_{t=0}^{74}R_t\prod_{h=1}^{t}\frac{(1+\lambda_h)}{(1+r_h)}+\prod_{h=1}^{74}\frac{(TF_{74})}{(1+r_h)}}_{\text{Present value of expenses}\approx\text{Liabilities for the valuation period}}\right]}\approx 1 \qquad \textbf{[20]}$$

**APPENDIX IV: THE ACTUARIAL BALANCE FOR THE CASE OF JAPAN**

The simplified expression of the Japanese actuarial balance using the terminology of the US model is:

$$\underbrace{\overbrace{TF_0}^{\text{Trust fund}}+\overbrace{y_0\sum_{t=0}^{94}\theta_t N_t\prod_{h=1}^{t}\frac{(1+g_h)}{(1+r_h)}}^{\text{Present value of contributions}}+\overbrace{\sum_{t=0}^{94}S_t\prod_{h=1}^{t}\frac{1}{(1+r_h)}}^{\text{Present value of subsidies}}}_{\text{Present value of incomes 2005-2100}}=\underbrace{\overbrace{B_0\sum_{t=0}^{94}R_t\prod_{h=1}^{t}\frac{(1+\lambda_h)}{(1+r_h)}}^{\text{Present value of benefits}}+\overbrace{\prod_{h=1}^{94}\frac{(TF_{94})}{(1+r_h)}}^{\text{Trust fund}}}_{\text{Present value of expenses 2005-2100}}$$

**[21]**

Where, $S_t$ : Amount by which the State subsidises the pension system in year t.

As already mentioned, this balance is compiled every five years taking into account a 95-year time horizon.