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The asset and management tool of a public agency

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The asset and liability management tool of a public agency

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CADES background

- A mission : CADES – Caisse d'Amortissement de la Dette Sociale - is the defeasance vehicle of French social security cumulative deficits

- CADES was initially granted a unique resource, a tax levied on the nation overall revenues called CRDS - Contribution au Remboursement de la Dette Sociale ; it was recently endowed with an additional resource outsourced from another tax, the CSG or Contribution Sociale Généralisée.

- Debt outstanding has been extended thrice since its birth in jan. 1996

- Until 2004, CADES life span definition was a fixed end date and its funding scheme was a fixed tax rate contribution. Having experienced both an extension of its life duration and an increase of the tax rate, the reference to a fixed end date was suppressed, and its funding scheme now allows an adjustable tax rate. Assessing the total debt's final time of repayment remains CADES responsibility.

Balance-sheet representation

Asset	Liability
>>> defined tax base	*>>> selection of debt instruments*
Income from Activity	Debt Instruments Portfolio
Income from Replacement Revenues	
Income from Assets and Estate	
Proceeds from Gambling and Jewellery	

- a mirror image of a managed fund balance-sheet

Balance-sheet representation

- The asset side : a defined benefits scheme could apply to Cades asset

  - The taxable base of the contribution received by Cades is defined,
  - and the contribution/tax rate is decided by the government and voted.

- The liability side : while initial debt outstanding is decided exogenously, the debt instruments are chosen according to the goal of the company – to achieve the mandate it was given, ie the reimbursement of an outstanding debt in an optimal way.

Modelling the economy

- The initial value of the contribution revenue once assessed, its evolution can be modelled by a real growth rate, and a rate of inflation. Let

  g_t, *real growth rate*

  i_t, inf *lation rate*

  and k_t *the* value *growth rate in* continuous - time *is the sum* $g_t + i_t$

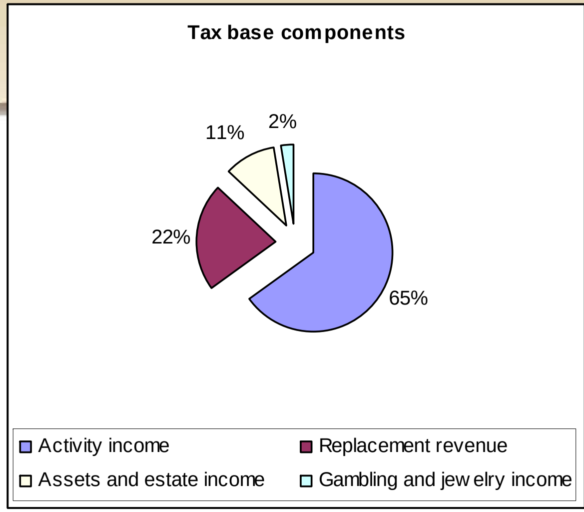
- The economy is also driven by the short-term interest rate. We assume its dynamic follows a Ornstein-Uhlenbeck process, along with the real growth rate and the inflation rate. We specify a Vasicek formulation of the zero-coupon rate curve. For instance the short-term interest rate evolution has the following differential equation :

$$dr_t = a(b - r(t))dt + \sigma_r dW_r(t)$$

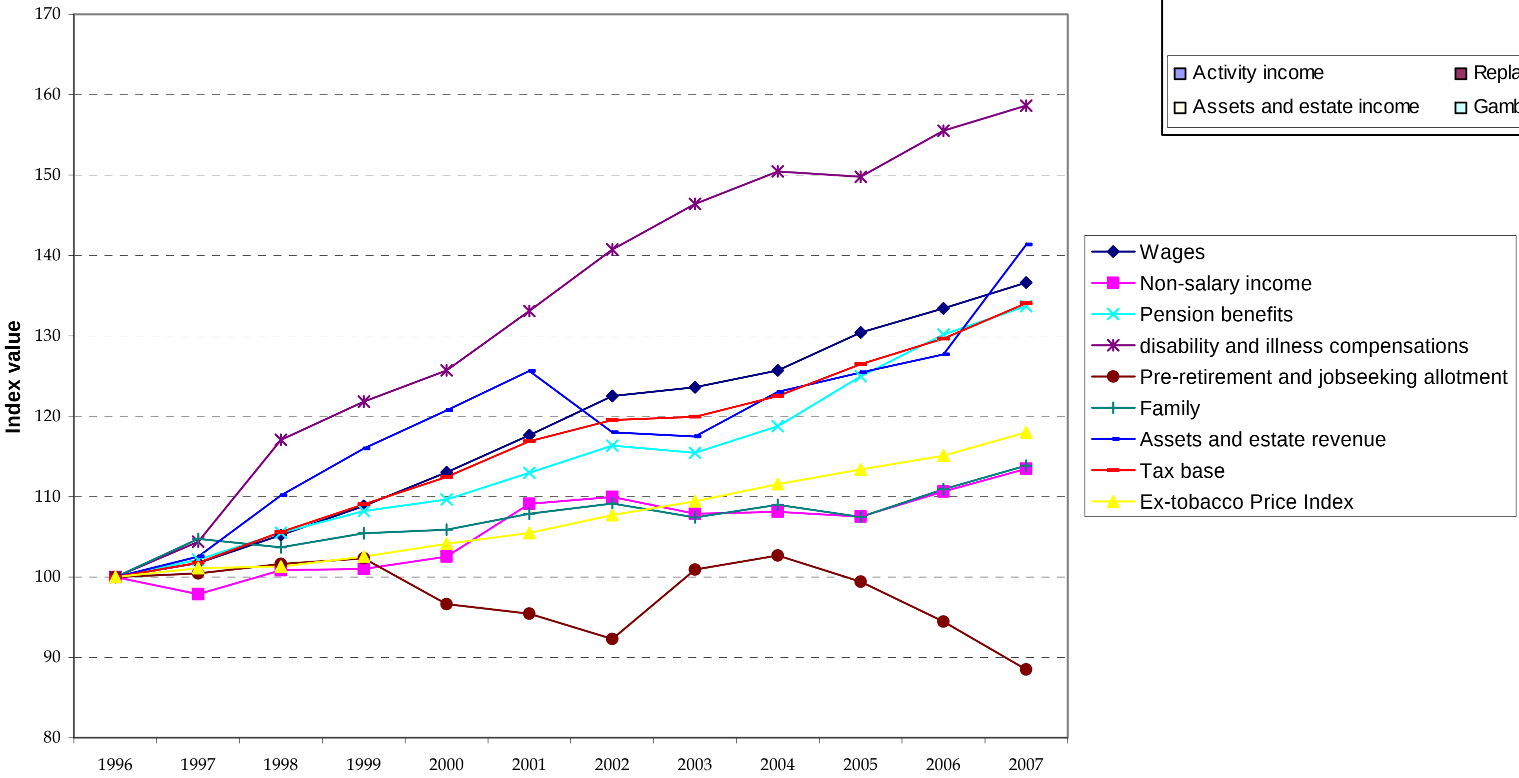
- Each of the three variables contains a source of risk modelled by a Wiener process ; the three Brownian motions are linked one-another by their cross-correlations. After some transformation one gets a system of three equations with uncorrelated sources of risk. For instance the inflation rate dynamic will be modelled by the following equation :

$$di_t = c(d - i(t))dt + \rho_{r,i}\sigma_i dW_r(t) + \sqrt{1-\rho_{r,i}^2}\,\sigma_i dZ_i(t)$$

Asset description



Growth index in real terms by category of revenue and Price Index (rebased at 100 in 1996)



Asset description : stylized facts

Tax base, GNP, and Price Index (rebased at 100 in 1996)

Balance-sheet modelling

- The asset A_t has the following dynamic

$$dA_t = A_t(g_t + i_t)dt$$

- Knowing a given liability amount L_t,

  - We compute the disposable as the resources/expenses balance.
  - From there, the net balance is attained by subtracting payables.
  - When positive we can further reduce the existing debt with buy-backs, otherwise a financing requirement is met with an issuance program. The value of the net debt annual variation will fluctuate with the cheap/dearness of the rates environment.

- The net debt dynamic being defined as,

$$L_t = L_{t-1} - CA_t$$

- with CA_t the amortization capacity at year-end t, it also reads

$$L_t = L_t^* - N_t$$

- with L_t* the debt value before reallocation,
  and N_t the net balance

Optimization program

- In the modeled world, the buy-back, as well as the issuance program, is structured according to a rule tantamount to the CPPI(*) rule. The goal is to maximize amortization capacity at any time t, so that our net debt may reach the null value at the best pace.

- Therefore the objective function can be written as,

$$\underset{\alpha}{\text{Min}}\ L_t^{\ +} = \left[\sum_k \sum_m \left(B(t,m) E_k(t,m) - \alpha(k,m) N_t \right) \right]^+$$

- where $E_k(t,m)$ is the outflow at maturity m for the k-th class of debt, as of time t,
- $B(t,m)$ the time t value of a risk-free zero-coupon bond maturing on year m,
- $\alpha(k,m)$ the allocation coefficient for maturity m and k-th class of debt,
- and for any variable X,

  $X^+ = Sup(X,0)$
- The amortization capacity is cumulated until net debt value is null, in other terms debt is extinguished at time s when we reach

$$L_s^+ = 0$$

(*) CPPI : Constant Proportion Portfolio Insurance

Optimization program : a dual form

- Indeed an error in the estimation of debt redemption horizon H seen from time 0 bears some consequences. Should such an error occur, CADES would have to refinance at time H the amount

$$L_H = L_{H-1} - CA_H$$

- Therefore, CADES faces the risk of an additional borrowing need, either met by its life extension, or by an extra tax raise.
- Let us label α this risk, the optimization problem can be re-written in a dual form such as :

$$\text{Min } \alpha = P\left[L_{H(\alpha)-1} - CA_{H(\alpha)} > 0\right]$$

- The time dependency of this risk reveals our risk aversion. For a given initial debt, the lower the amortization capacity, then for a same capacity decrease, the greater the reimbursement period lengthening.

- Seen from the tax payer standing point, this looks like a put selling position, for his loss is growing with the magnitude of the estimation error.

Program solving

- The adopted re-balancing rule is the CPPI method. The solution is a (k x m) size vector of allocation α *(k,m)*.

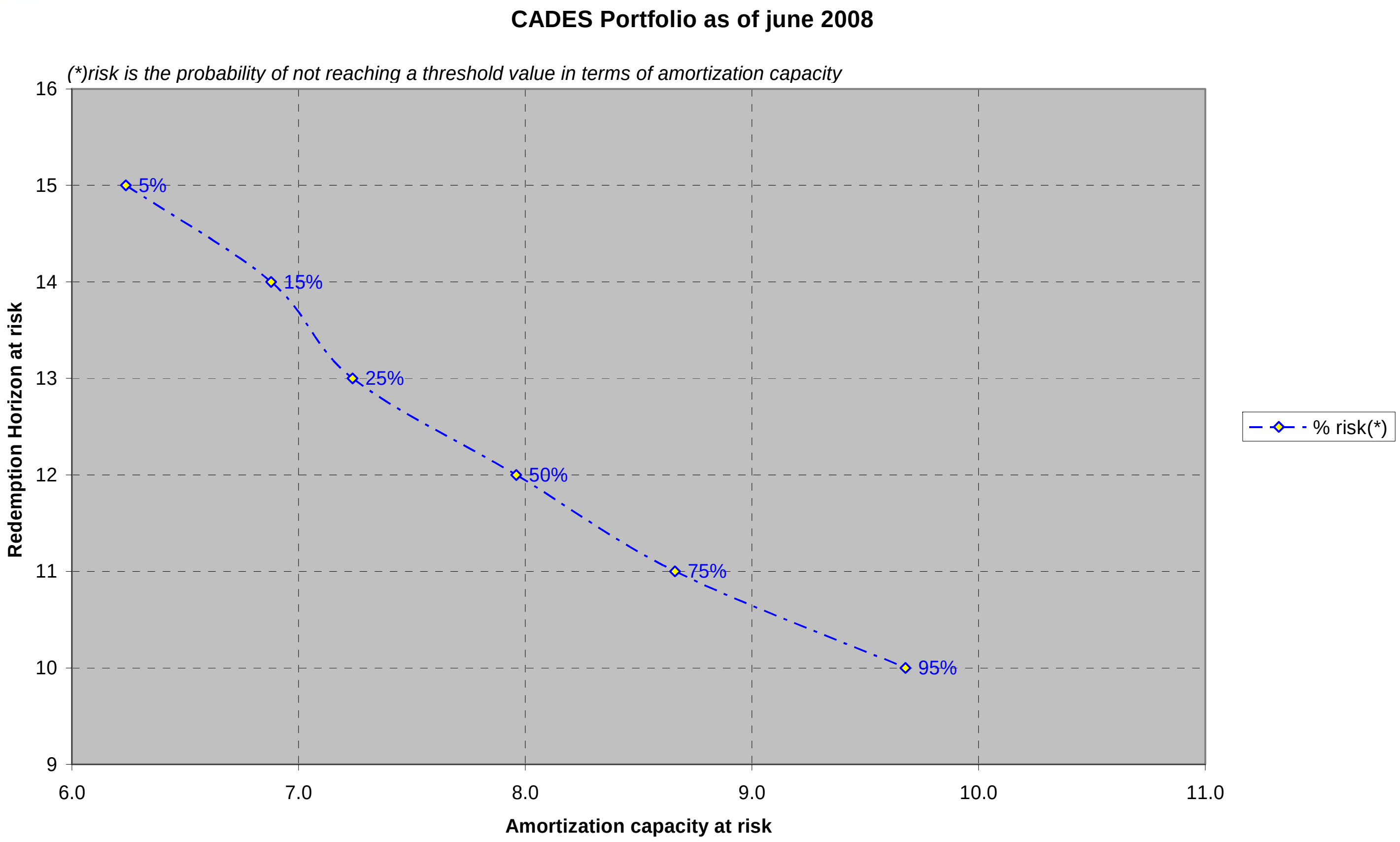
- We do not solve numerically the optimization problem.

- Rather, we simulate a reasonable number of portfolio structures

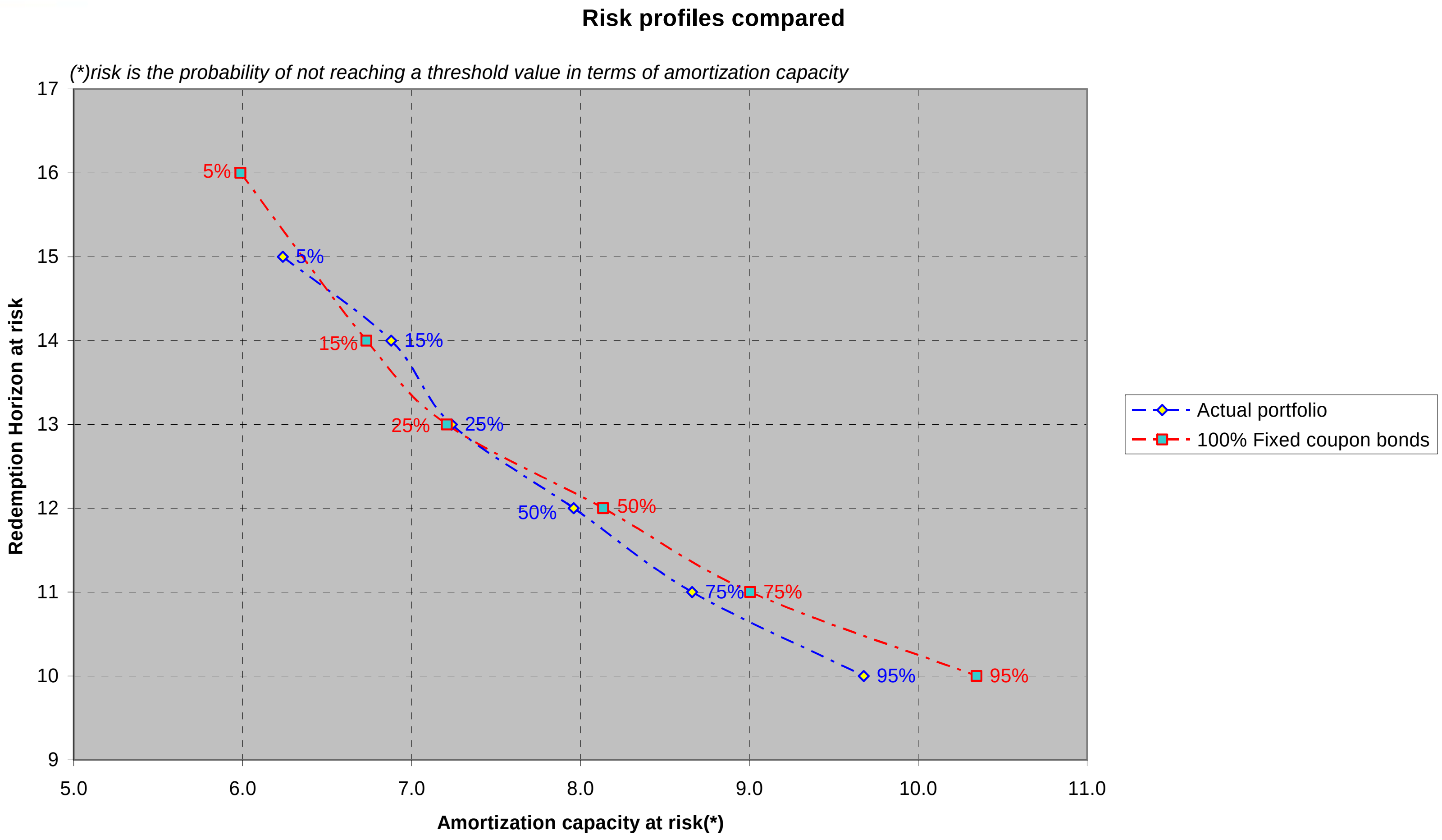
  - remarkable structures called « extreme » portfolios, each one composed with a unique class of debt,
  - structures built by allowing to switch 10% of a portfolio between two debt classes, on the basis of the current debt portfolio

- We represent the universe of portfolios in a two-dimension space, bearing performance and risk on the two axis. We can rank portfolios along an axis called direction of enhancement, and exhibit the orthogonal direction of no-arbitrage, namely an axis which bears portfolios with no trade-off in terms of performance vs risk arbitrage.

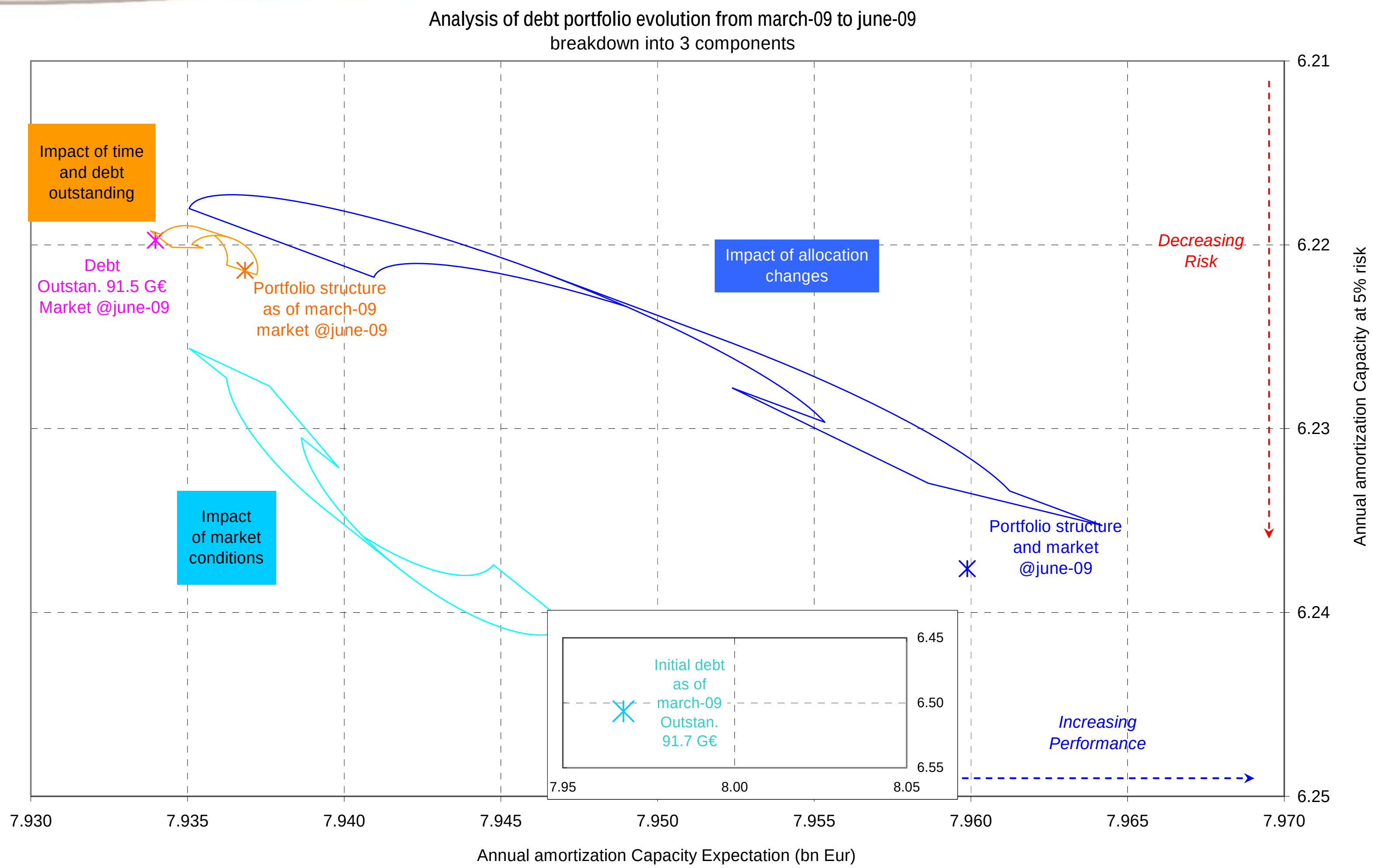
Debt risk profile



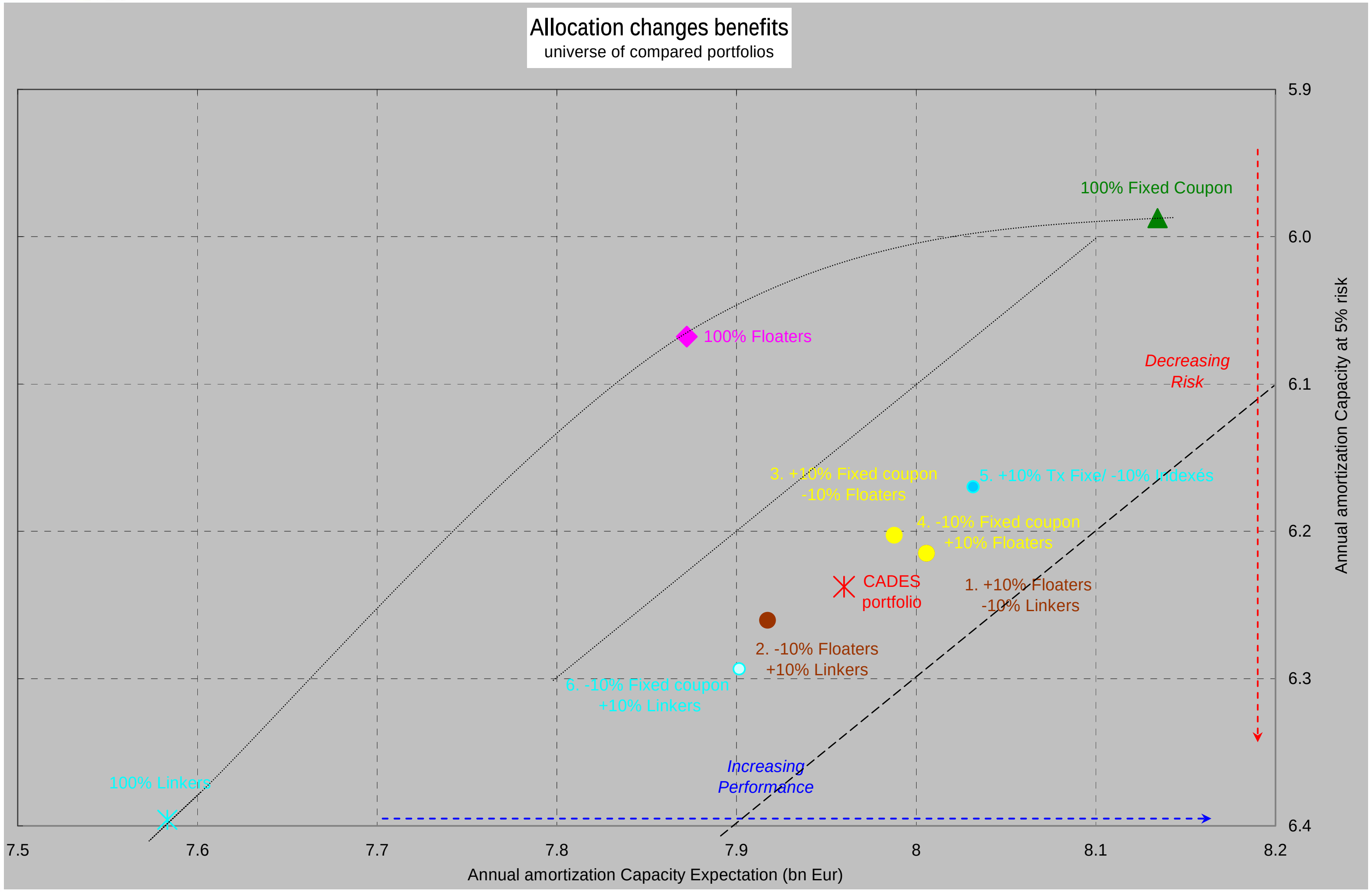
Stochastic dominance



Dynamic analysis on the actual portfolio



Static arbitrage of portfolio structures

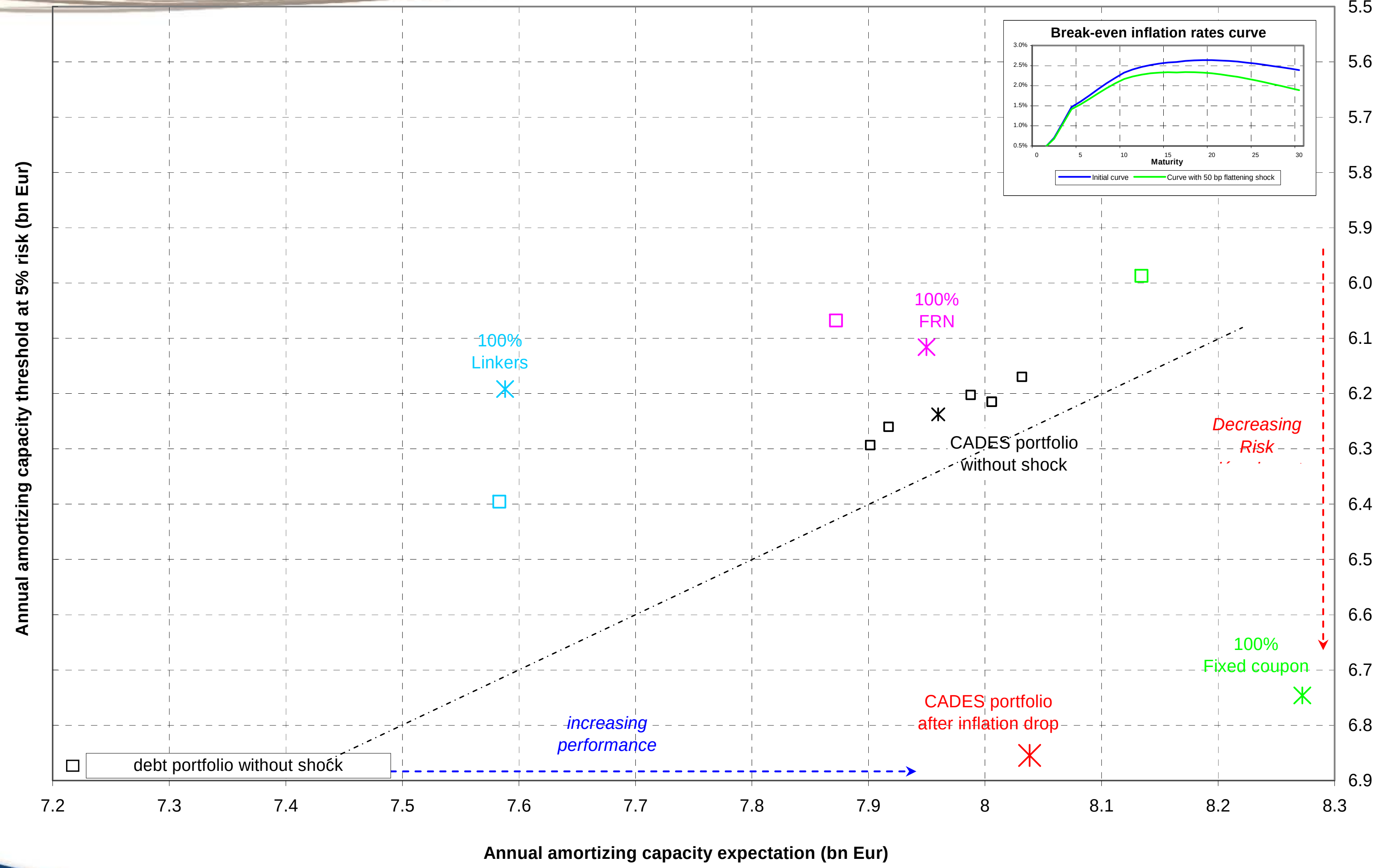


Sensitivity to shocks : exploring model risk

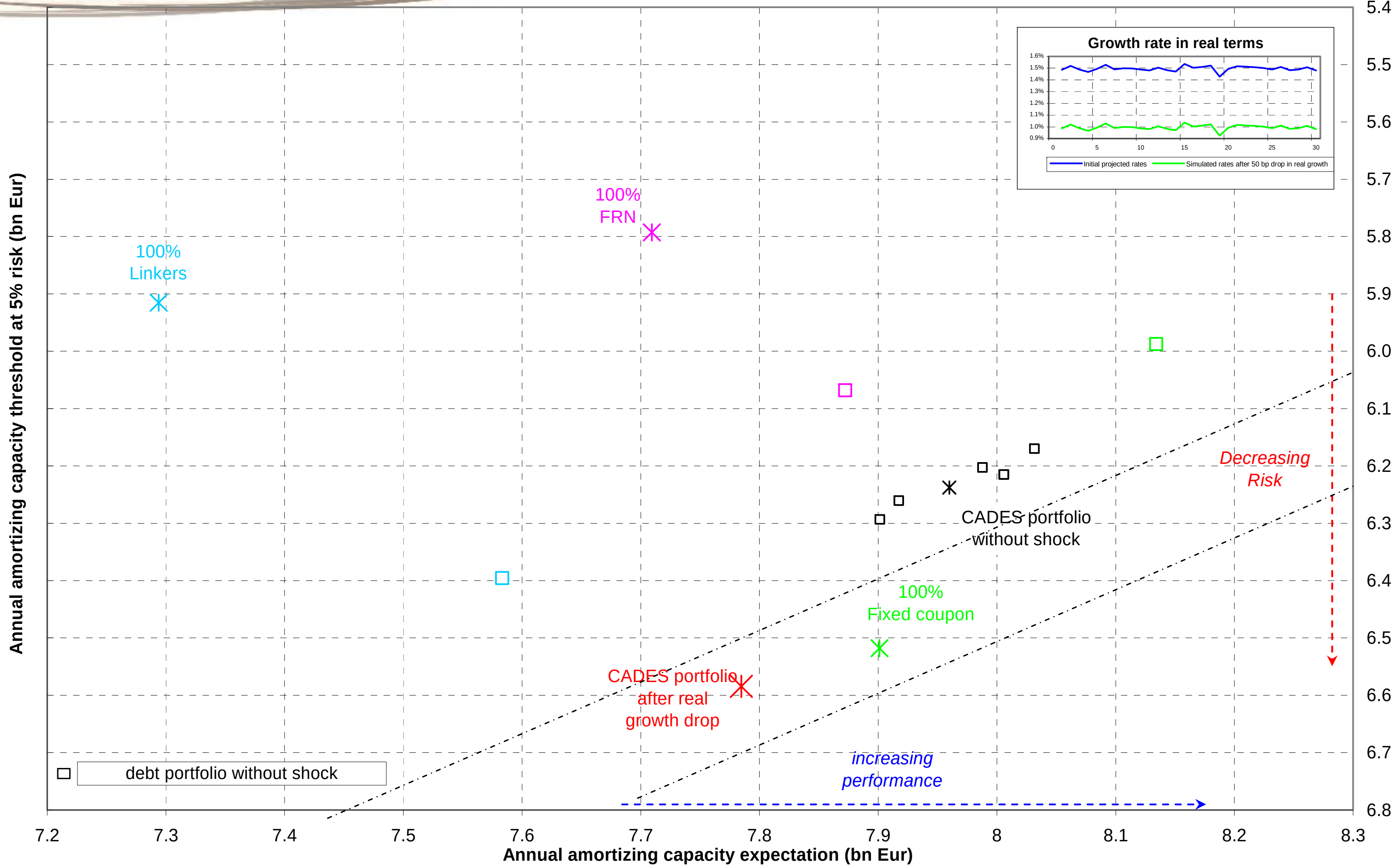
Sensitivity to shocks affecting initial conditions		annual amortization capacity			redemption horizon		
		at 5% risk(1)	median	at 95% risk(1)	at 5% risk(2)	median	at 95% risk(2)
CADES debt as of June-09		**6.24**	**7.96**	**9.68**	**15**	**12**	**10**
parallel move of nominal yields curve	+100 bp	-0.52	-0.50	-0.47	2	1	1
	-100 bp	0.58	0.49	0.39	-1	0	0
slope of nominal yields curve	+100 bp	0.45	-0.00	-0.36	-1	0	1
	-100 bp	0.89	0.42	-0.14	-1	0	0
slope of break-even inflation rates curve	+50 bp	0.83	0.43	-0.03	-1	0	0
	-50 bp	0.62	0.08	-0.35	-1	0	1
revenues growth	-50 bp	0.35	-0.18	-0.65	0	1	1

(1) risk of staying shy of the threshold value
(2) risk of staying above the threshold value

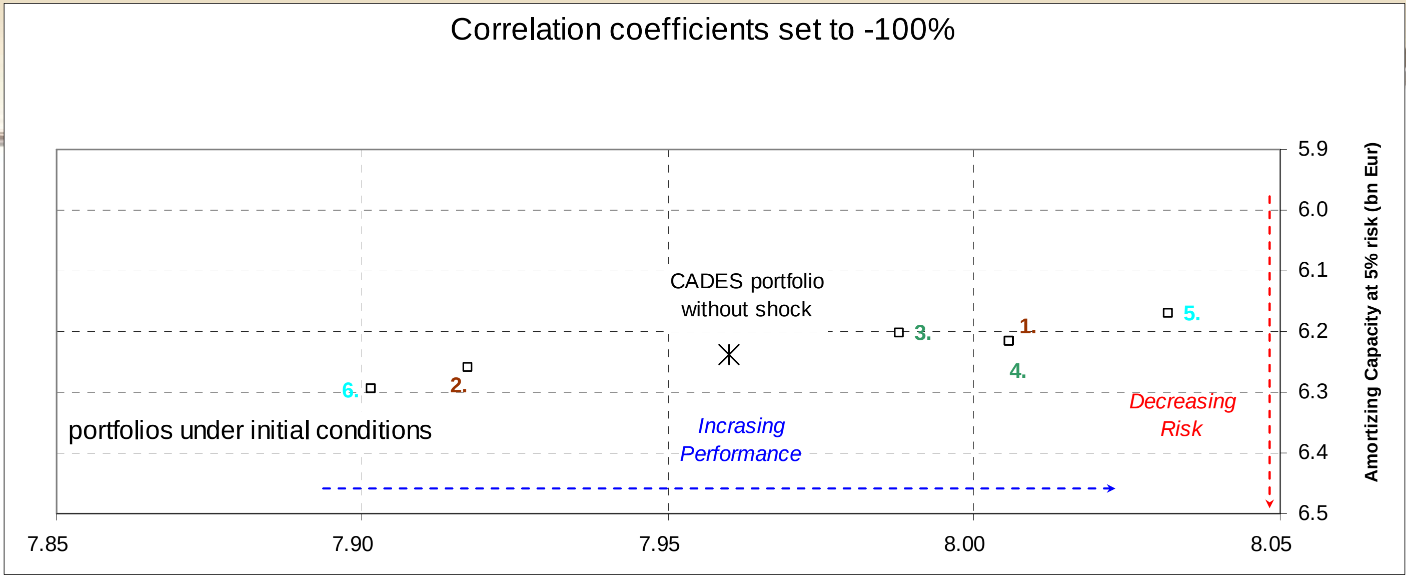
Sensitivity to shocks: flattening the expected inflation rates curve

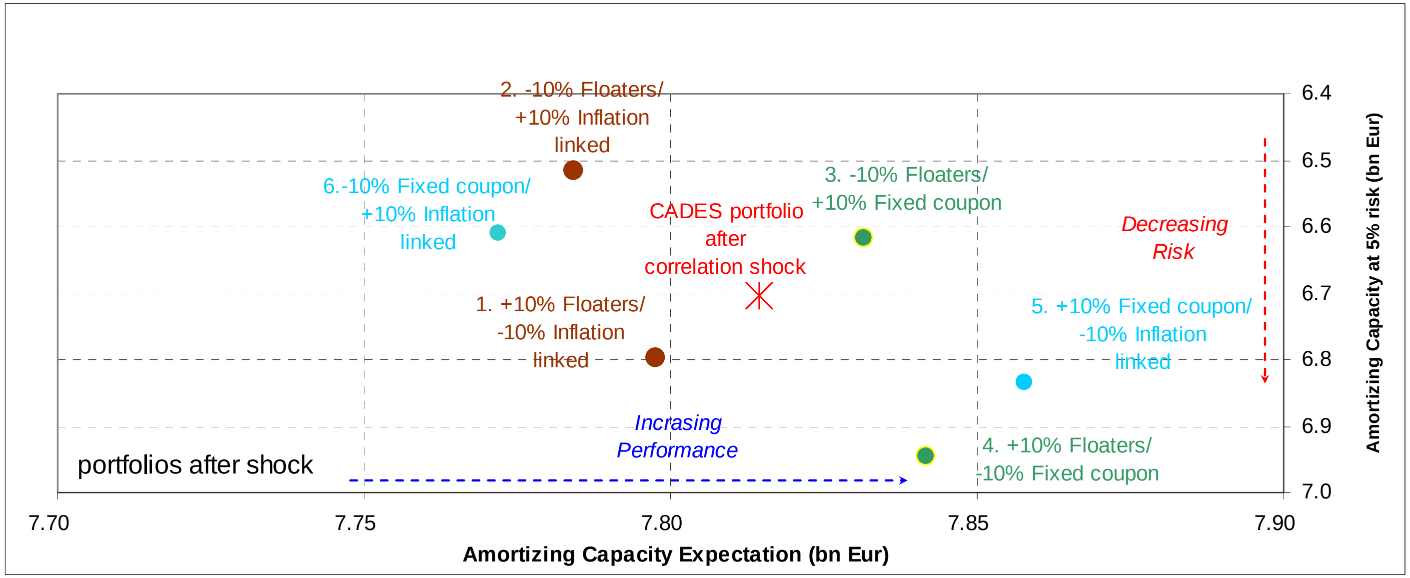


Sensitivity to shocks: depressing the drift of real growth rate



Sensitivity to shocks: stressing the correlation matrix





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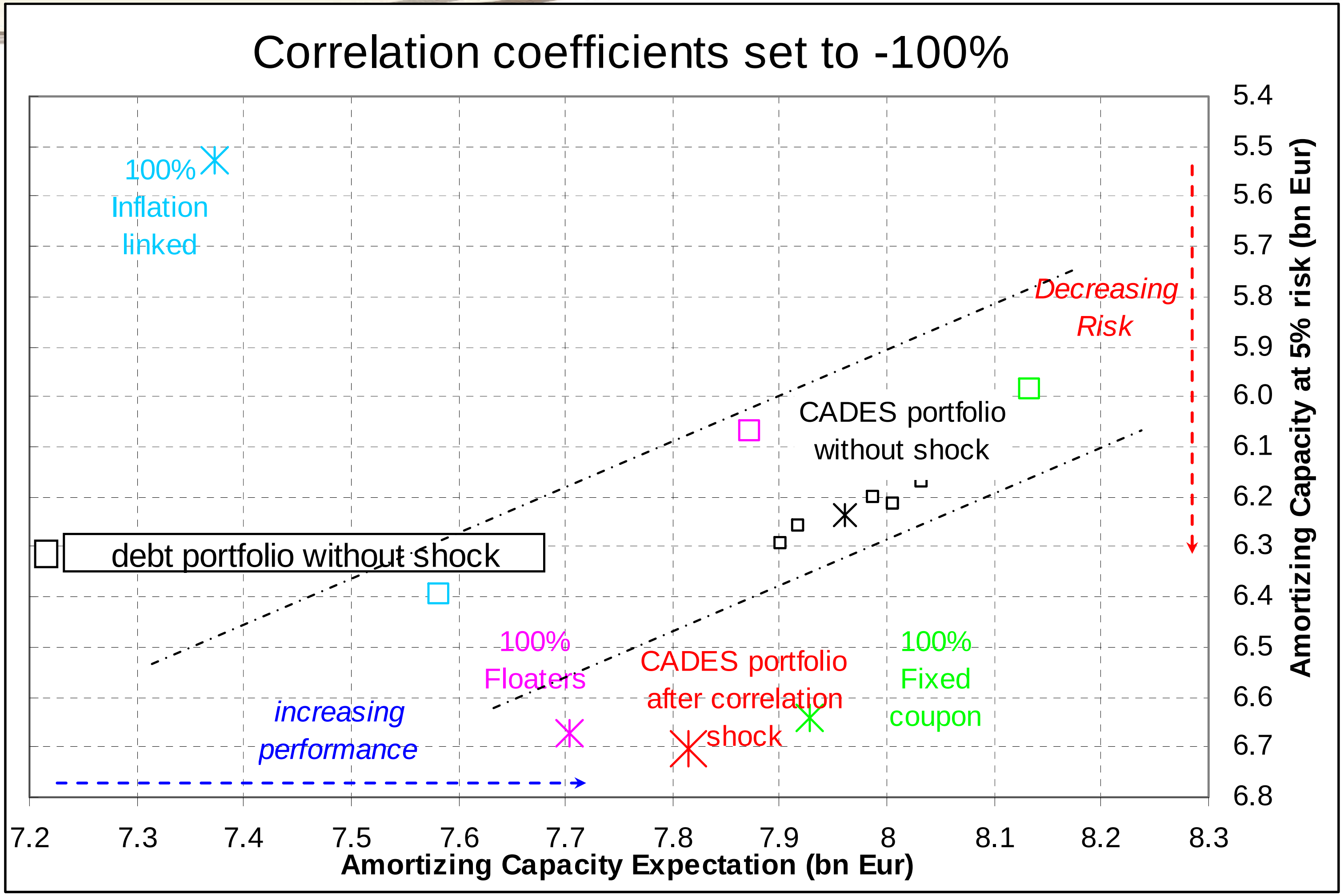
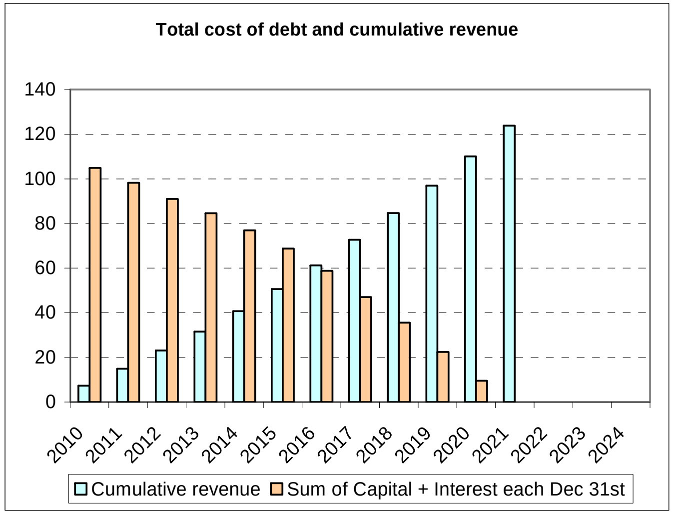
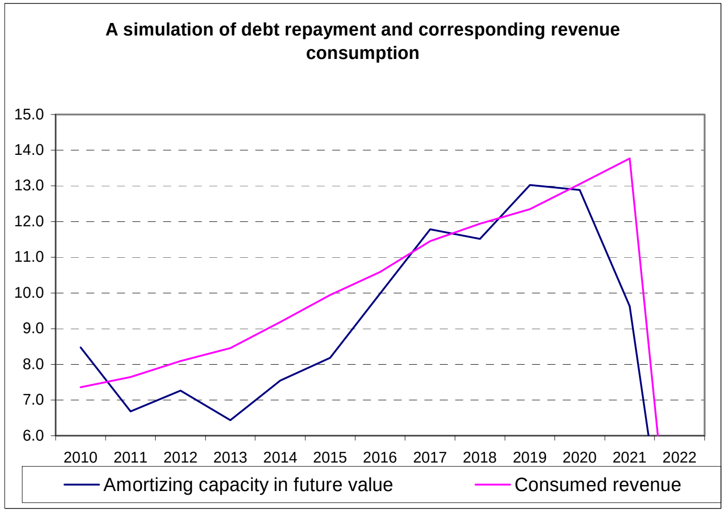


Sample of debt repayment

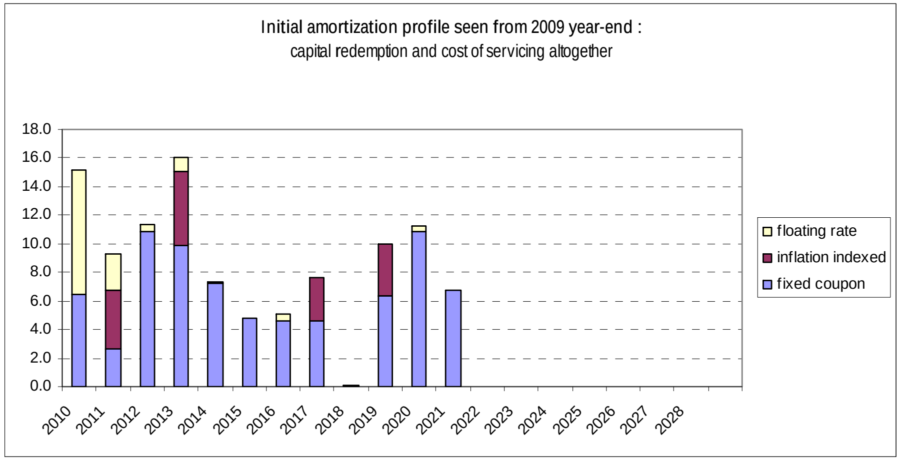
Initial net debt oustanding (bn Eur)	91.65
Additional debt transfer	0.0

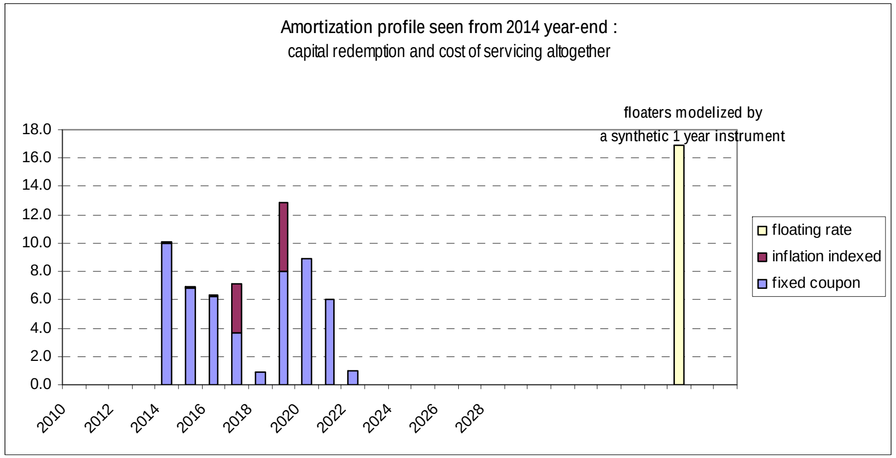
Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
Sum of Capital + Interest each Dec 31st (bn Eur)	113.42	104.94	98.26	91.00	84.56	77.01	68.83	58.85	47.06	35.54	22.51	9.63	0.00	0.00	0.00	0.00	0.00
Amortizing capacity in future value		8.48	6.68	7.26	6.44	7.55	8.18	9.98	11.78	11.52	13.03	12.89	9.63	0.00	0.00	0.00	0.00
Consumed revenue		7.36	7.64	8.10	8.46	9.18	9.95	10.59	11.45	11.94	12.35	13.05	13.77	0.00	0.00	0.00	

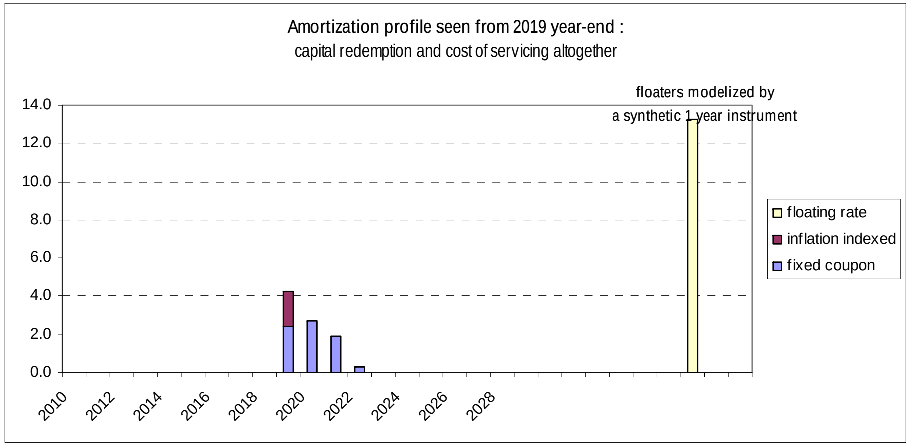




Sample of debt repayment : portfolio projections







Foreseen enhancements

- Comparison with the results of a VAR model
- Change the step of the model from yearly to quarterly or half-yearly
- Introduce other financial measures, such as IRR
- Introduce a time-dependent allocation vector in the optimization program, solve with Jacobi-Bellman