The simulation of a deficiency on a plan termination basis in defined benefit pension plans

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Abstract

In Japan, many Tax Qualified Pension Plans (TQPPs) will shift to the new corporate pension plans such as Defined Contribution (DC) plans and Defined Benefit (DB) plans and many other TQPPs will be terminated due to the abolishment in March, 2012. And the introduction of the new accounting standard for the post-employment benefits in 2000 has also worked to increase DC plans. Furthermore the regulatory burden of DB plans may be one reason why DC has increased. And this tendency is strengthened when the investment environment becomes unstable.

However, I think it leads to problems to shift to DC plans easily in the following circumstances of our country.

・Most TQPPs are used to pay benefits as the part of the retirement lump sum plans and the employers have always taken responsibility for the post-employment benefits for the purpose of assuring the long-term service of talented employees.

・It is hard to think that the employees who are almost amateurs of the investment can invest well in DC plans when even the expert can't do well in DB plans.

One of the main reasons why the employers avoid DB plans and select DC plans is likely that they can't forecast how much money will be necessary in the worst case.

In particularly unstable circumstances in which the DB plan sponsors can't help terminating the plans, their responsibilities to pay the benefits balanced to past services are heavy. Therefore it is important for them to simulate a deficiency on a plan termination basis in the future.

Such a simulation is not popularly used except in several large plans in our country but it may be more important in small plans.

This paper spotlights the idea of the simulation but does not describe its techniques in detail.

Preface

In Japan, many Tax Qualified Pension Plans (TQPPs), defined benefit corporate pension plans, are going to shift to other approved corporate pension plans such as Defined Contribution (DC) plan and Defined Benefit (DB) plan. (*1) Rest of TQPPs will be terminated before the deadline, March, 2012. Among 44 thousand terminated TQPPs from April, 2002 to September, 2008, 10% of them shifted to DC plans, 10% to DB plans, 30% to the Smaller Enterprise Retirement Allowance Mutual Aid (SERAMA) and 50% terminated completely.

Introduction of the new accounting standard for the post-employment benefits in 2000 affected plan sponsors to choose DC plans.(*2) On the other hand the regulatory burden of DB plans would also be another reason why the number of DC plans have increased. This tendency is strengthened when the investment environment is unstable.

Many employers seem to shift TQPPs to DC plans for avoiding the fluctuation of net periodic pension cost and/or contribution to DB plans under dull business environment. However, it should not be easily accepted to shift TQPPs to DC plans or abolish them in the following circumstances.

・The retirement lump sum defined benefit plans have developed good relationship between employers and employees.

・The post-employment benefits are well-paid for long-term service for the purpose of retaining talented employees. (In DC plans success in asset investment allow them get enough return)

・It is hard to expect that employees who are almost amateurs for asset investment can get enough return in DC plans while even experts cannot do well in DB plans. In addition, sum of investment costs for DC plan participants may be higher than those of DB plans.

・Most employers adopted TQPPs for tax incentives instead of securing lump sum severance payment.

・Employees have to pay heavy tax on distributed money at TQPPs termination.

What is necessary for encouraging employers to shift TQPPs to DB plans under the coming aging population when post-employment benefit plans become more important?

One of the main reasons why the employers do not select DB plans would be that they cannot forecast how much money will be necessary in the worst scenario.

In case that the DB plan sponsors cannot help terminating plans, their responsibilities to pay benefits balanced to past services would be heavy. Under the regulation of DB Law (Defined Benefit Corporate Pension Law) it is necessary for plan sponsors to pay the amount of the deficiency on a plan termination basis immediately. (refer to section 2) And those amounts would be large when plan termination funding level is low.

Therefore it is important for DB plan sponsors to simulate the possible future deficiency. Particularly small DB plan sponsors might need such simulations because they have more factors to fluctuate the deficiency than those of big plans. However such simulation is not popular with DB plans for now and the necessity of such simulation has rarely been argued because it is troublesome work and takes much cost.(*3)

With the aim of relieving uneasiness of DB plan sponsors this paper focuses the spotlight on some ideas of the simulation while I do not describe techniques or results in detail.

It would be grateful if this paper would open up discussions and be referred to plan designing and administration .

#All the opinions expressed in this paper are that of the author alone and do not necessarily reflect the views of his employer or JSCPA.

(*1) Suppose that DB plans and DC plans in this paper are based on DB Law (Defined Benefit Corporate Pension Law) and DC Law (Defined Contribution Pension Law) in Japan. I do not mention EPFs(Employee's Pension Funds) which are other kind of defined benefit plan.

(*2) TQPPs and DB plans may shift to DC plans more due to income fluctuation when International Accounting Standards require plan sponsor to recognize actuarial gains and losses immediately in income statement.

(*3) It is necessary for DC plan sponsors to explain how post-employment benefits will fluctuate based on asset investment result.

1. The merits of DB plans in our country

Responsibility for defined benefit at retirement in order to retain talented persons for long have grown lump-sum severance payment plan nationwide. And TQPPs have taken a role to provide a part or all of lump-sum severance payment. Such a concept of lump-sum severance payment plan is different from that of DC plans because DC plans pay money for every period of employees' services and employers do not take responsibility for defined benefit at retirement.

Toward TQPPs' abolition of March, 2012, TQPPs are shifting to DB plans and/or DC plans at an increasing pace now. A lot of TQPPs have shifted to DC plans because TQPPs' cost might largely fluctuate under unstable environment. The trend from TQPPs to DC plans seems to continue also in the future.

With aging population and slow economic growth in current Japan, we cannot expect much for the public pension system. Therefore corporate pension plans are hoped to grow as the preparation for post-employment. And DB plans are seemed to be main corporate pension plans as they have following merits compared with DC plans.

- Large scale investment brings higher efficiency and lower administration cost.
- Employees can concentrate in their work because they do not need to think about asset investment.
- Employees can receive almost the same amount of post-employment benefit as they expected.
- No investment education cost on employees are needed for employers.
- Employers can cut lump sum benefit in the case of employees' voluntary withdrawal.

In addition employees would prefer DB plans to DC plans as they tend to favor cooperation over competition.

2. The reason why many employers do not select DB plans

Employers seem to avoid DB plans due to following reasons.

(1)Impact on cost of DB plans in financial statement after introduction of new accounting standard for post-employment benefits

New accounting standard for post-employment benefits introduced in 2000 requires plan sponsors to recognize unfunded amount of PBO within defined years. Since then it made financial statements worse as the discount rate in PBO became lower and investment return on assets turned worse. Furthermore, employers are concerned about serious impact on financial statement when International Accounting Standards require recognition of unfunded amount immediately.(*4)

(2)Two strict rules, plan continuation basis and plan termination basis, in comparison with TQPPs

The regulations of DB Law require plan sponsors to pay contribution for the purpose that they do not only continue DB plans but also keep minimum assets when they terminate those plans.

- Plan termination basis

In our country, withdrawn participants from DB plans enjoy benefits, but participants are not vested for past service benefits. Therefore plan termination basis was introduced in DB Law so that enough assets would remain for participants to receive their benefits at plan termination.

Under current unstable environment it might be happy for participants that DB plans can continue but it is also important for them that plans have enough assets to pay amounts for total past services.

This basis sets minimum funding amount (hereinafter referred to as MF) which plan sponsors should secure for each participant and pensioner at plan termination.(*5) And it requires plan sponsors to pay amounts of plan deficiency below MFs (=total amounts of MF) at DB plan termination.

(3) Incomprehensible and troublesome feature in managing DB plans

Some employers avoid DB plans as they feel incomprehensible and troublesome in managing DB plans because of above mentioned bases. So DC planners might give them the impression that DB is difficult to manage.

Have we ever explained enough about such management to employers? Have we explained enough how deficiency at plan termination will fluctuate with various cases in the future and how they can manage to reduce such fluctuation? I myself often reflect on this matter.

Can we make plan sponsors feel secure when they design or manage DB plan by explaining such fluctuation? Because of similarity with lump-sum severance payment plans, employers would select DB plan rather than avoid it if they know range of the fluctuation in the future. It is miserable for employers to be required to pay the amount of the deficiency at termination after they design DB plans without knowing such fluctuation. Furthermore it is also miserable for participants if employers terminate plan due to uncertainty of DB plans.

(*4)Some Japanese pension actuaries study how to introduce hybrid pension plan with characteristic of DB plan and DC plan, which may also reduce fluctuation of unfunded amount.

(*5)Simulations described later define MF as lump sum benefits based on voluntary withdrawal or the amount paid when participants request lump sum benefits after they select pensions at withdrawal, which are the amount accumulated at interest rate for deferral before payments and the balances after payments which is accumulated at interest rate in replacing lump sum benefits with pension. Be careful that this definition is near to the definition of regulations of DB Law in Japan but isn't same. The regulations of DB Law in Japan allows plan sponsor to set MF lower than lump sum benefits based on voluntary withdrawal. In this definition participants might leave before serious situation because lump sum benefits are bigger than MF which distribute at DB plan termination and it leads further deficiency.

3. Factors affecting plan deficiency below MFs

One of the reasons why employers avoid DB plan is the fluctuation of net period pension cost and/or contributions to plan. Particularly they cannot imagine the amount of additional contributions at plan termination with plan deficiency below MFs though they worry about how large those contributions will be at the worst condition. Therefore it is important that we explain not only factors for the fluctuation of the deficiency but also figures of the simulation of the deficiency to employers.

【Factors affecting plan deficiency below MFs 】

Main factor of the fluctuation of deficiency is investment returns on plan assets. The deficiency may fluctuate widely in short term if plan assets are invested in stocks or foreign bonds while it would fluctuate narrowly if they are invested in domestic bonds or life insurance products (*6) (Figure-1). It also leads to deficiency that actual investment return rate is always less than assumed interest rate in calculating plan contribution even if investment return rate hardly fluctuates.

DB plans often fall into deficiency even if investment return rate is always equal to assumed interest rate. One reason is the shape of benefit curve drawn depending on service years. (Figure-2) Many Japanese lump-sum severance payment plans pay backloaded benefits, so that MF (=lump sum benefit) is bigger than normal costs accumulated at an assumed interest rate for long service years and smaller for short service years. This kind of benefit curve is called S-shaped curve. DB plans fall into deficiency when participants leave after longer service years more than assumed withdrawal in calculating plan contribution. They may also fall into deficiency when actual salary increase is higher than assumed salary increase in final salary plans.

Another reason is withdrawals under lack of plan assets accumulated in the past. Under severe business environment plan sponsors often aim at high risk/return investment in order to lower contribution to DB plans, which could cause enormous loss. When plan assets are less than plan MFs, benefit payments to withdrawal participants cause to expand deficiency ratio for remaining participants.(Figure-3) So it would bring miserable results to remaining participants.

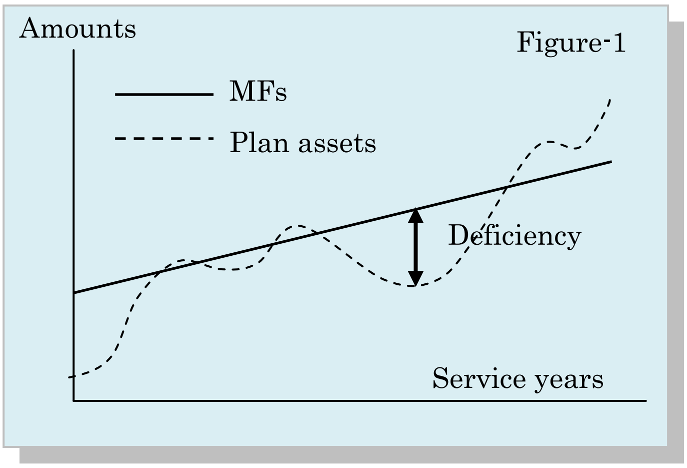


Figure-1

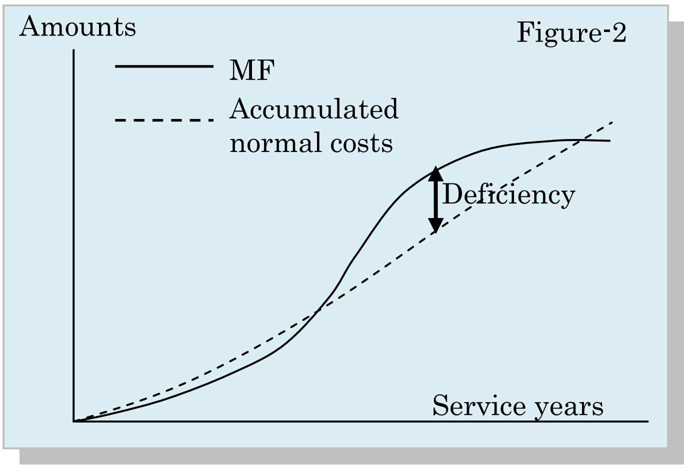


Figure-2

Figure-3

	Before withdrawal	Payment to withdrawal	After withdrawal
① MFs	10,000	2,000	8,000
② Assets	6,000	2,000	4,000
③ ②／①	60%	-	50%

(*6)Some Japanese insurance companies provide the investment product named GA (General Account) to pension plans, which guarantee fixed interest rates.

4. Simulation of plan deficiency below MFs and use of the result

It is important for plan sponsors to know future funding level in various scenarios at designing DB plans as if we confirm quake resistance from several points of view at designing a house.

Final salary pay DB plans with S-shaped curve tend to fall deficiency below MFs (Figure-2). If small plan sponsors design DB plans which pay all benefits replacing lump-sum severance payment plans with zero percent assumed withdrawal rate at old age because of no experience in the past(*7), many withdrawals at old age by restructuring would immediately increase the deficiency. Plan sponsors could have changed the plan design if they had known this impact on the deficiency. It might be one-sided view to focus on only plan deficiency because they can reduce personnel expenses burden thanks to withdrawal. However they would be able to run DB plans at ease if they know such impact.

It is almost true that plan sponsors and the participants do not have expertise about financial management of DB plans. Plan sponsors might feel DB plans untrustworthy if deficiency increase immediately after they founded DB plans with no advice about them. Furthermore they cannot avoid covering deficiency if they terminate plans.

If they want to continue those plans, it is important for them to design or review their plans after they know the impact from the deficiency. If they want to raise the probability that plan assets exceed plan MFs, it is a shortcut for them to pay additional contribution. However they could also realize that by reviewing plan asset allocation and/or the shape of benefits curve. They feel more secure if DB plans' contributions are set with confirming future deficiency. After all it would lead to build confidence among members concerned with DB plans that plan sponsors have an interest in the plan termination funding level (ratio for plan MFs against plan assets), disclose funding level and simulate the fluctuation of the deficiency periodically.

(*7) Assume that 5% of one hundred participants withdrew from DB plans in past three years. Then the following inequality shows that 95% credible interval of withdrawal rate is 2.5%～7.5%.

$$\hat{p} - u(\frac{\varepsilon}{2})\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leqq \text{credible interval} \leqq \hat{p} + u(\frac{\varepsilon}{2})\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\hat{p} = 5\% \qquad \varepsilon = 100\% - 95\% \qquad n = 100 \times 3$$

$$u(x) = \{x \,;\, \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{x^2}{2}}\,\mathrm{dx} = \frac{\varepsilon}{2}\}$$

How credible is contribution calculated with withdrawal rate of 5% in small DB plans?

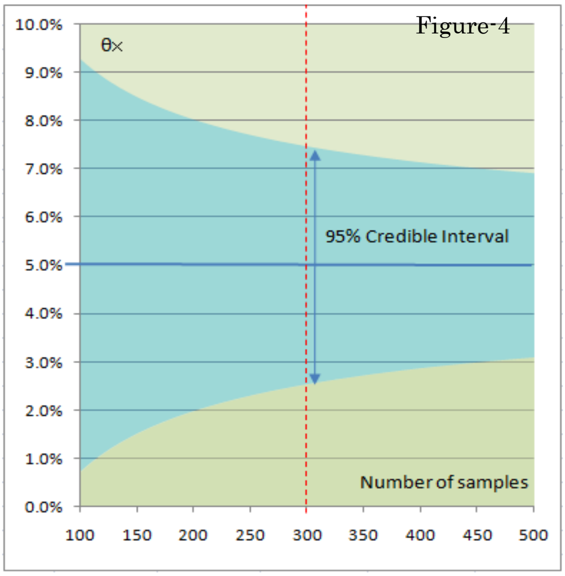


Figure-4

5. Simulation of plan deficiency below MFs

Fluctuation of plan deficiency below MFs in some period occurs from the difference between increase of total amounts of MF (benefit accrual in that period) and total contributions with actual investment return. Therefore it is clear that there is no fluctuation in DB plans where benefit is decided as sum of contribution and actual investment return.(Figure-5) However more or less fluctuation occurs because benefits are often decided based on S-shaped curve (and in the first place DB Law prohibits benefit linked with actual investment return). (Figure-6) It is misfortune for DB plan sponsors not to know how the deficiency will fluctuate in the future. However it is meaningless if they cannot understand it and/or do not have willingness to understand

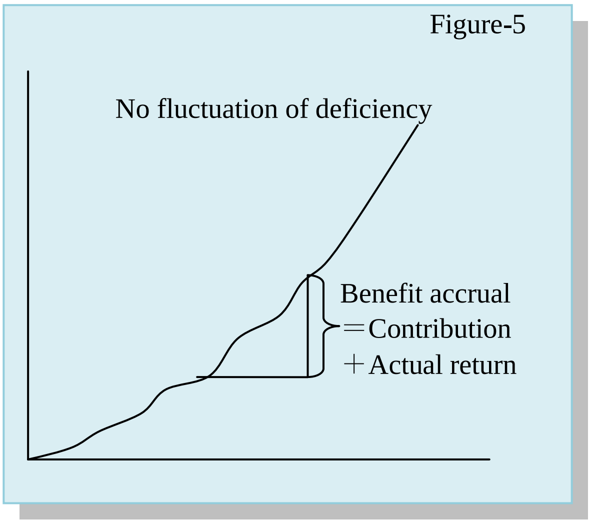
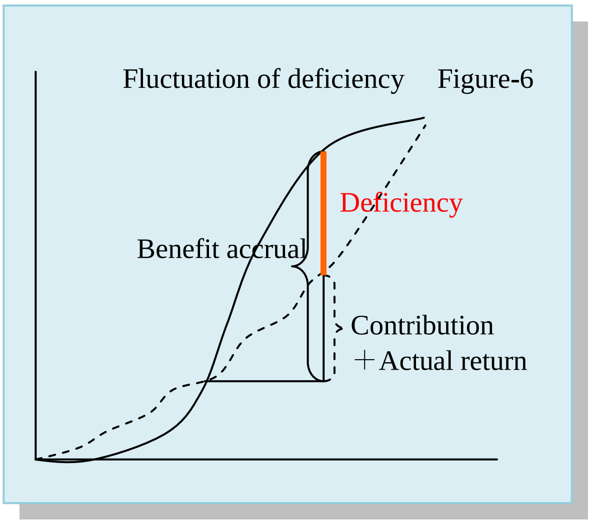


Figure-5

it.

Therefore how about showing the result of deficiency simulation from the following view points?

・When DB plan sponsors design plans and/or change assumptions
・Using probability distribution
・Using computer graphics to show them visually



And how about providing two kinds of deficiency simulations as follows to help plan sponsors to understand how to design DB plans and how to set additional contribution to avoid fluctuation of deficiency?

Simulation1

Simulating difference between MF and accumulated contributions when participant leaves or retires at certain age with certain service years by using certain assumptions. (*8)

Simulation2

Simulating difference between plan MFs and plan assets in the future by using Monte Carlo simulation with expected value of several random variables. In case of using assumptions based on past experience in small DB plans, it is appropriate to set allowance for credible interval (*7) of the assumption.

Those simulations use several benefit formulas and experience assumptions. Different assumptions lead different results. Particularly small plan sponsors should not depend on the result itself excessively because of low credibility of assumptions. Rather it would be more important that they take notice on the difference resulting from the difference of benefit formulas, asset investment portfolios and actuarial assumptions. By using those results they can consider plan design and/or asset investment.

Several kinds of assumptions necessary in simulations are as follows. (*8)

・Benefit formula
  ・Final salary pay, Average salary pay, certain percent of salaries accumulated with certain interest rate
  ・Pension types, annuity certain, whole life annuity (with guaranteed period)
  ・Retirement age and pension start age
  ・Reduction ratio for benefit in the case of participant's voluntary withdrawal, including no reduction
  ・Interest rate in replacing lump sum benefit with pension benefit when participant chooses pension in place of lump sum benefit
  ・Deferred interest rate on pension benefit in deferral period
  ・Interest rate used in accumulating certain percent of salaries, minimum guaranteed interest rate and the interest rate deducted from actual return rate in CB plan
・Funding ratio for plan MFs
・Actual return on asset investment portfolio
  Expected actual return rate μ, Expected standard deviation σ in Simulation2
・Timing of benefit payment and contribution accumulated
・Actuarial assumptions about the plan participants of DB plan
  ・Attribute of participants, for example, sex, current ages, birthday, past service

・Probability of mortality, probability of withdrawal, probability of selecting lump sum benefit, probability of salary increase

・Actuarial assumptions used in calculating contribution

・Assumed interest rate, mortality rates, withdrawal rates, salary increase rates, new entrants

It is important that assumptions used in those simulations are disclosed and the results of those simulations are shown clearly. For that purpose the results of those simulations should be shown not only with numerical values but also with visual graphics. Furthermore they should also be shown with average, standard deviation and the number of paths where plan deficiency below MFs become more than certain amounts in Simulation2.

Those simulations would be able to help plan sponsors to design DB plan and to set its contribution which deficiency is hard to occur.

(*8) The assumptions in simulation examples described later are shown in Appendix. And assume that all assumptions are independent one another though it might not be so, for example relation between "improvement of investment environment" and "salary increase more than expected", relation between "salary increase less than expected" and "increase of withdrawal".

(1) Simulation 1

This is the simulation to show how current ages, past services and withdrawal ages of model participants affect the deficiency, the difference between MF and contributions accumulated. And how different the results change with assumptions, for example benefit formula, reduction ratio in voluntary, assumed interest rate, expected actual return rate, salary increase and so on.

Simulation1 shows the deficiency at each withdrawal age on the assumption that new employees take part in DB plan immediately. The result of them is shown in matrix $\left(d(x_0,\tau_0,x_w)\right)$ such that $d(x_0,\tau_0,x_w)$ is the following function with current age x_0, past services τ_0 and withdrawal age x_w.

$$d(x_0,\tau_0,x_w) = B^W(x_0,\tau_0,x_w) - \left\{ B^W(x_0,\tau_0,x_0)\times(1+i^A)^{x_w-x_0} + \pi \sum_{t=0}^{x_w-x_0-1} S(x_0,t)\times(1+i^A)^{x_w-x_0-t} \right\}$$

such that $B^W(x_0,\tau_0,x)$ is the expected benefit based on benefit formula when the model participant who is at age x_0 with past service τ_0 leaves at age x. π is the contribution rate as normal cost. $S(x_0,t)$ is the expected salary at t-years later. i^A is the expected actual return rate.

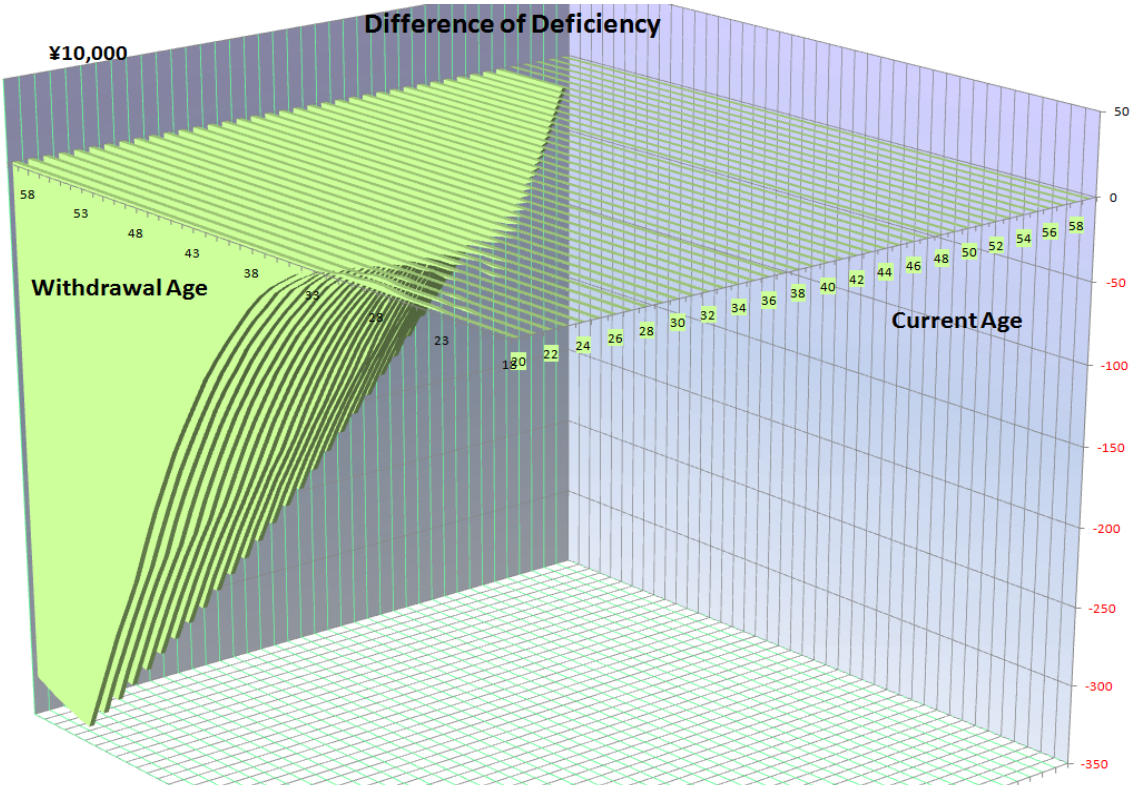
Several examples about difference of the deficiency are led by several assumptions in following table.

No.	Benefit formula	Reduction ratio in voluntary	Assumed interest rate	Expected actual return rate	Fluctuation of salary
1	Final salary pay	Yes	2%	2%	expected
2	Final salary pay	Yes	2%	2%	expected+30%
3	Final salary pay	Yes	3%	2%	expected
4	Final salary pay	No	2%	2%	expected
5	CB plan	No	2%	2%	expected
6	CB plan	No	2%	3%	expected
7	CB plan	No	2%	2%	expected+30%

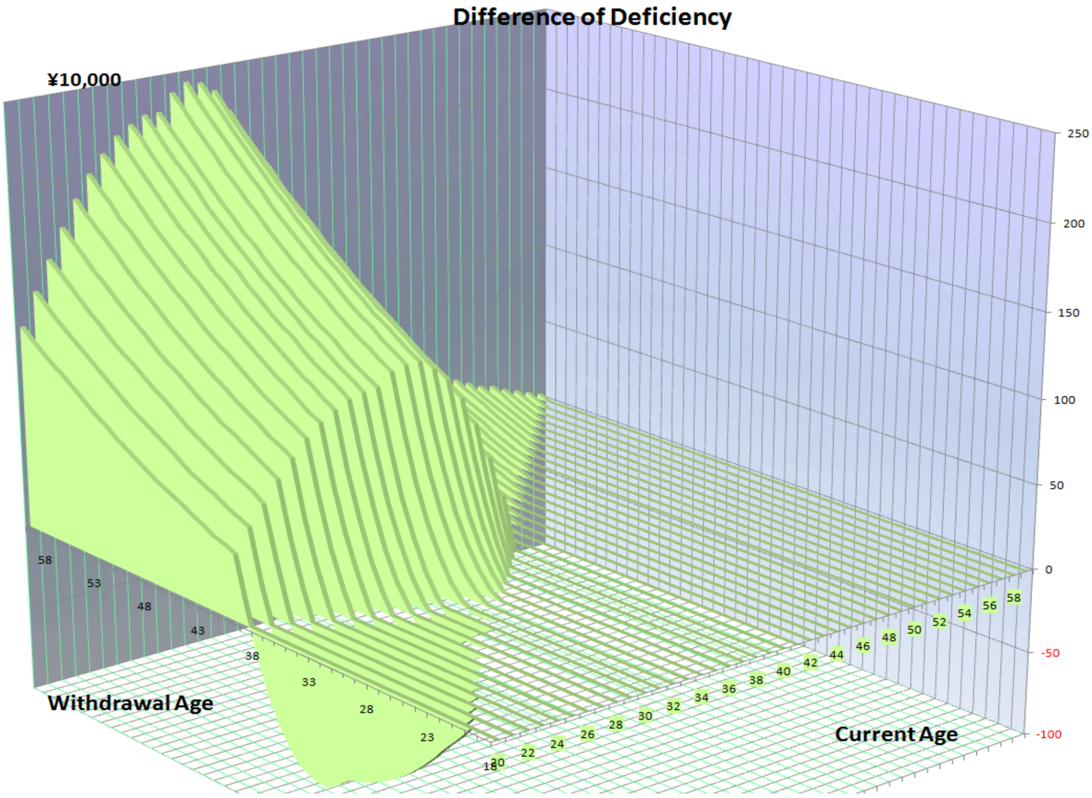
It might be convenient in considering plan design and/or asset investment that elements of comparison in simulation are shown as above table.

【Examples of disclosure about difference of the deficiency】

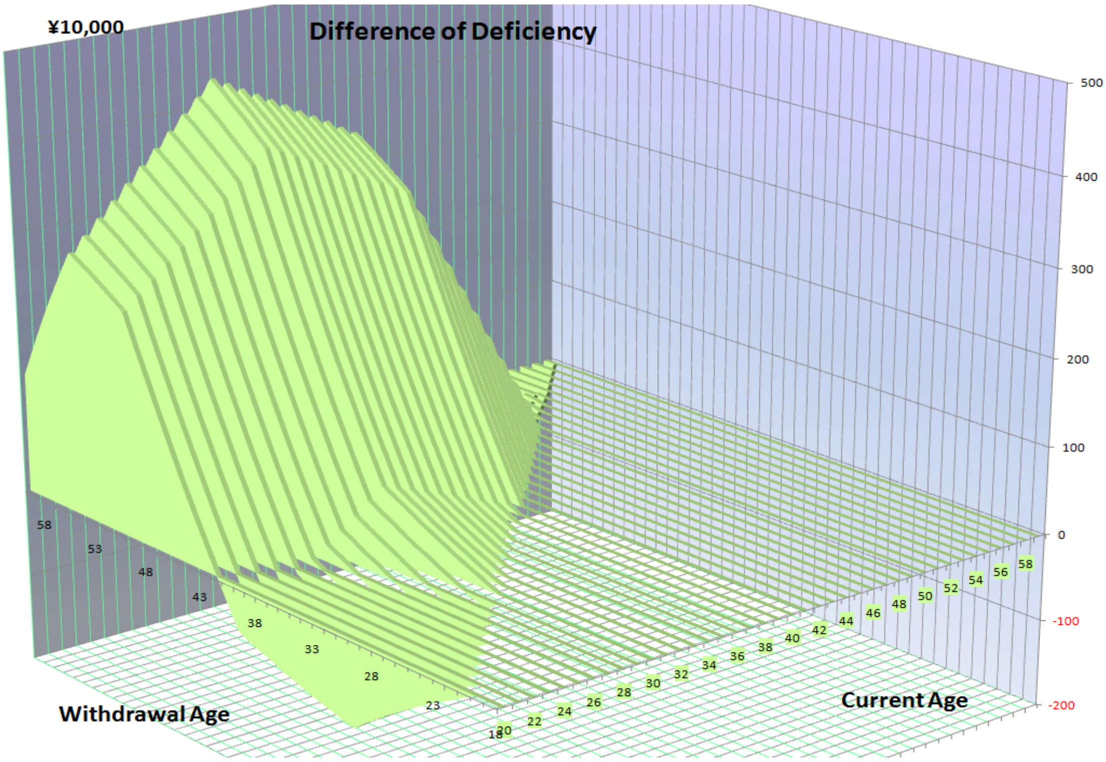
- By the salary increase rates in final salary pay plan　(No.2 - No.1)



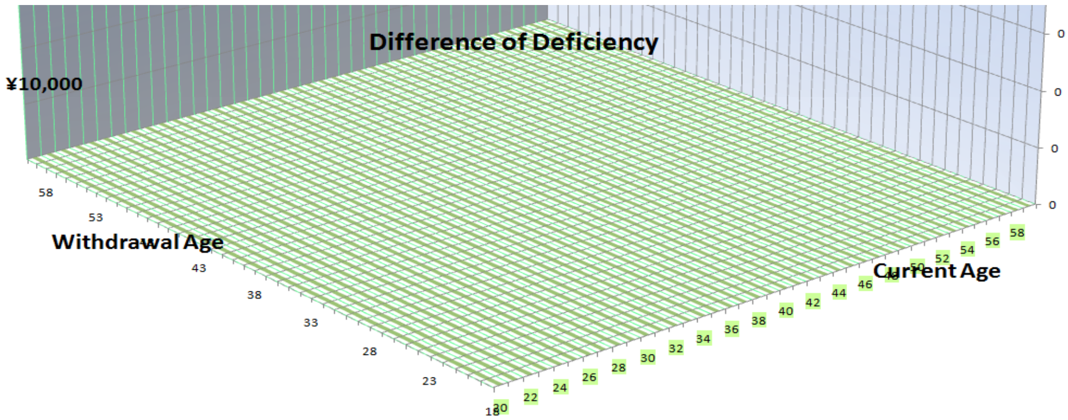
- By the reduction ratio in voluntary withdrawal in final salary pay plan　(No.4 - No.1)



- By the benefit formula though benefits at retirement age are same (No.5 - No.4)



- By the expected actual return rate in CB plan (No.6 - No.5)
- By the salary increase rates in CB plan (No.7 - No.5)



(2) Simulation 2

This is the Monte Carlo simulation for expected value of random variable. The plan deficiency below MFs are shown as distribution of expected values and as their mean and standard deviation.

In several plans with different benefit formulas or actuarial assumptions it is shown that distribution of expected values of the deficiency vary in future even if those values are same at the starting point (i.e. t=0).

Furthermore we can perhaps recognize the difference of expected values of the deficiency between DB plan with a “higher of” option and DB plan with no option though we can hardly recognize the difference in generally used actuarial valuation.

Several examples of disclosure about difference of the distribution of expected deficiency are led by several assumptions in following table.

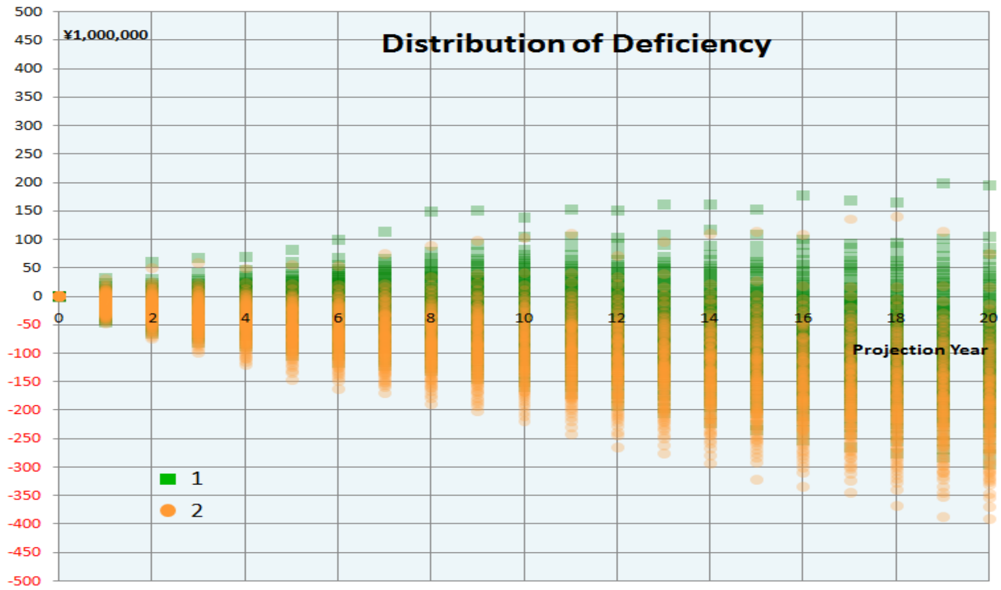
No.	Benefit formula	Reduction ratio in voluntary	Minimum guaranteed interest rate	Assumed interest rate	Expected actual return rate (μ,σ)	Fluctuation of salary	Probability of selecting lump sum benefit
1	Final salary pay	Yes	-	2%	2%,2%	expected	100%
2	Final salary pay	Yes	-	2%	2%,2%	expected+30%	100%
3	Final salary pay	Yes	-	2%	3%,5%	expected	100%
4	Final salary pay	Yes	-	3%	2%,2%	expected	100%
5	Final salary pay	Yes	-	3%	2%,2%	expected	0%
6	Final salary pay	No	-	2%	2%,2%	expected	100%
7	CB plan	No	-	2%	2%,2%	expected	100%
8	CB plan	No	-	2%	2%,2%	expected+30%	100%
9	CB plan	No	-	2%	3%,5%	expected	100%
10	CB plan	No	1%	2%	3%,5%	expected	100%
11	CB plan	No	1%(*)	2%	3%,5%	expected	100%
12	Final salary pay	No.1 but using the lower side of 95% credible interval of withdrawal rates					
13	Final salary pay	No.1 but using the upper side of 95% credible interval of withdrawal rates					
14	CB plan	No.7 but using the lower side of 95% credible interval of withdrawal rates					
15	CB plan	No.7 but using the upper side of 95% credible interval of withdrawal rates					
16	Final salary pay	No.1 but 50% withdrawal rate at 55 years old					
17	CB plan	No.10 but σ =7%					

(*) Accumulated rate is determined as "actual return rate – 1%"

In above simulations, iteration in the Monte Carlo simulation for expected value of random variable is 1,000 times.

【Examples about difference of expected distribution of deficiency】

- By salary increase in final salary pay plan (No.1 v.s.No.2)

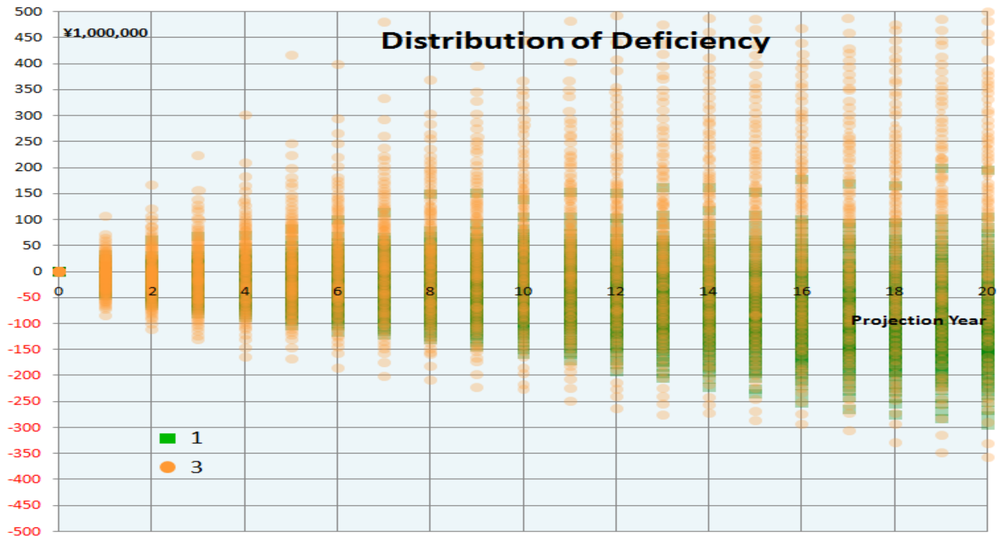


Deficiency of Assets for plan MFs and Contribution rate which deficiency will be 0 after 5 years later

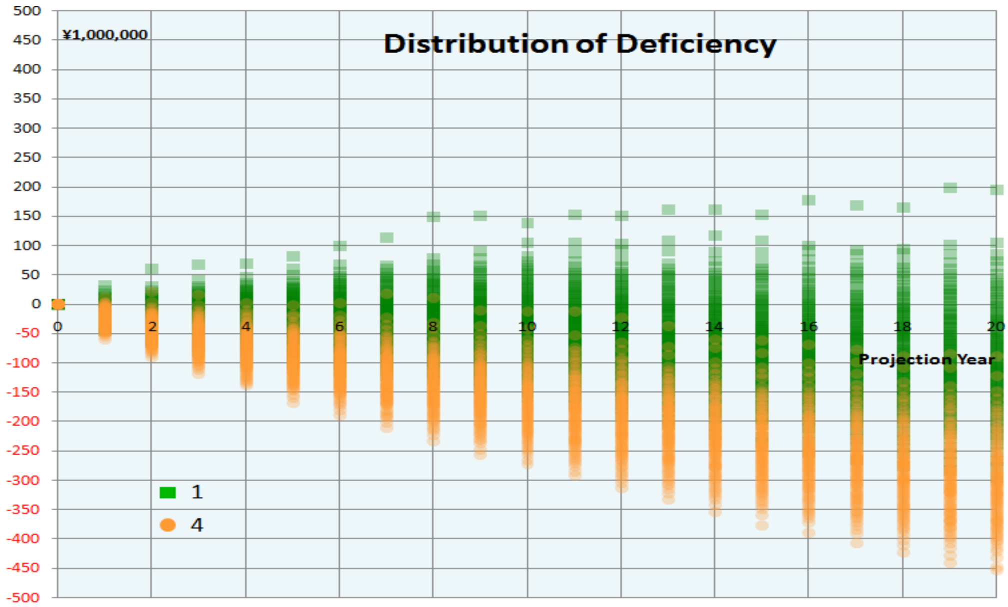
Mill. Yen

	case	Projection Year																				
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Average	1	0	-14	-20	-23	-25	-26	-28	-30	-32	-35	-37	-44	-51	-59	-65	-72	-78	-84	-89	-94	-99
	2	0	-14	-25	-37	-49	-59	-67	-76	-83	-90	-97	-108	-120	-131	-142	-153	-163	-172	-181	-189	-197
	1 − 2	**0**	**-0**	**4**	**14**	**24**	**33**	**39**	**46**	**51**	**55**	**60**	**64**	**69**	**72**	**77**	**80**	**85**	**88**	**91**	**95**	**98**
St Deviat	1	0	15	20	26	30	36	40	44	48	53	58	61	65	69	72	75	79	81	84	87	89
	2	0	15	22	28	32	36	40	45	49	53	57	61	64	67	70	74	77	79	83	84	87
	1 − 2	**0**	**-0**	**-2**	**-2**	**-2**	**-0**	**0**	**-1**	**-1**	**0**	**1**	**0**	**1**	**2**	**2**	**2**	**2**	**2**	**1**	**2**	**2**
Max10%	1	0	5	6	12	14	20	25	31	35	35	42	36	35	32	31	26	25	24	21	25	22
	2	0	5	3	-1	-9	-13	-15	-18	-21	-23	-23	-31	-42	-47	-53	-56	-61	-66	-71	-79	-84
	1 − 2	**0**	**-1**	**3**	**12**	**23**	**32**	**41**	**49**	**56**	**58**	**65**	**67**	**77**	**79**	**83**	**83**	**86**	**90**	**92**	**103**	**107**
Min10%	1	0	-31	-46	-56	-62	-73	-82	-88	-95	-103	-110	-121	-133	-145	-156	-170	-177	-185	-194	-201	-210
	2	0	-33	-53	-73	-90	-104	-120	-134	-147	-157	-169	-183	-201	-216	-229	-247	-260	-273	-287	-300	-310
	1 − 2	**0**	**1**	**7**	**16**	**28**	**31**	**38**	**46**	**52**	**54**	**58**	**62**	**68**	**70**	**73**	**76**	**83**	**88**	**94**	**98**	**100**
Number of the paths where >50	1	0	7	62	138	211	255	297	340	378	390	425	481	533	571	590	626	651	670	675	701	715
	2	0	8	121	333	465	593	663	719	746	766	788	840	868	889	906	911	917	930	940	946	943
	1 − 2	**0**	**-1**	**-59**	**-195**	**-254**	**-338**	**-366**	**-379**	**-368**	**-376**	**-363**	**-359**	**-335**	**-318**	**-316**	**-285**	**-266**	**-260**	**-265**	**-245**	**-228**
Number of the paths where >100	1	0	0	0	0	4	12	31	54	78	115	138	184	227	286	325	369	398	439	475	501	523
	2	0	0	0	7	63	124	222	309	373	435	479	547	629	681	739	772	805	819	837	852	865
	1 − 2	**0**	**0**	**0**	**-7**	**-59**	**-112**	**-191**	**-255**	**-295**	**-320**	**-341**	**-363**	**-402**	**-395**	**-414**	**-403**	**-407**	**-380**	**-362**	**-351**	**-342**

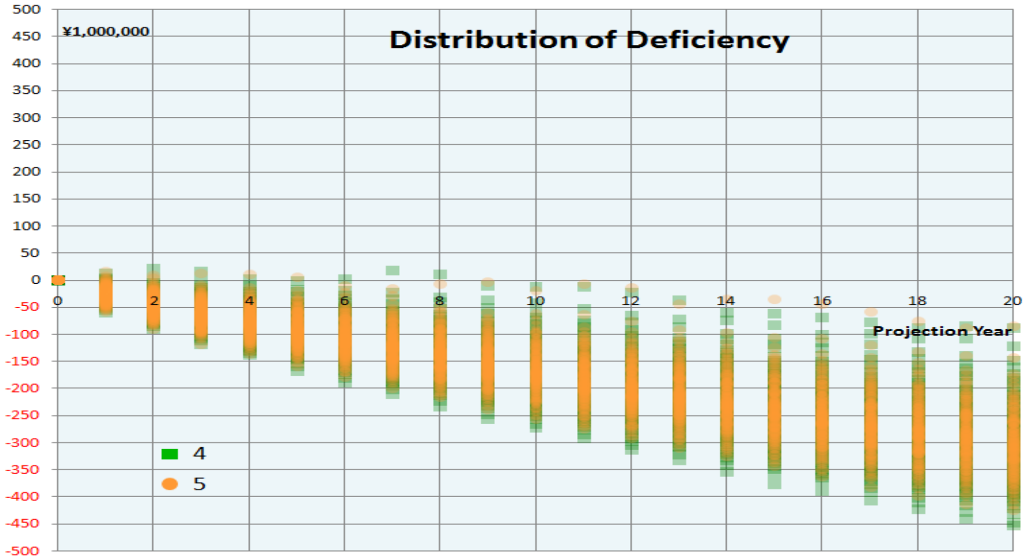
- By actual return rate in final salary pay plan (No.1 v.s.No.3)



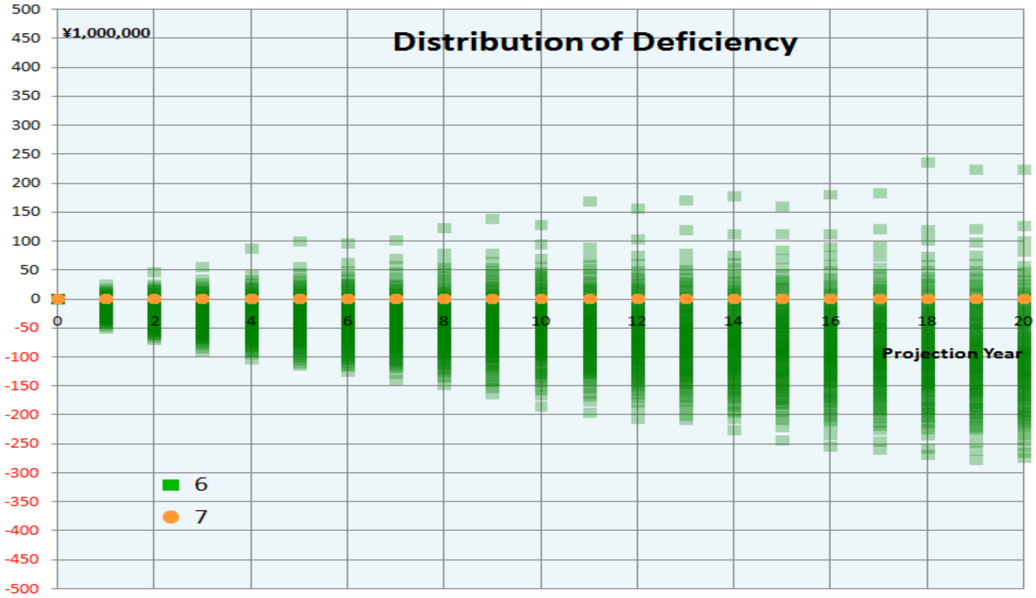
- By assumed interest rate in final salary pay plan (No.1 v.s.No.4)



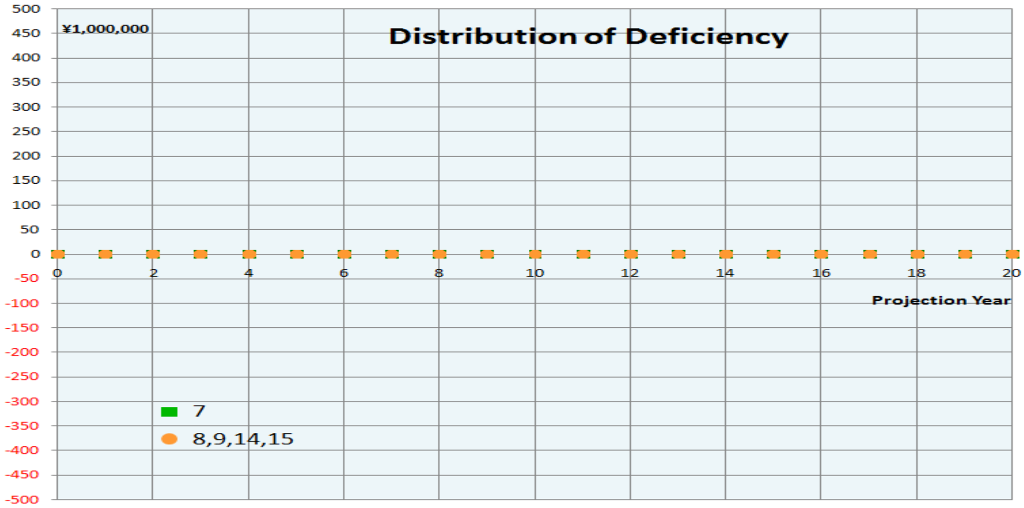
- By probability of selecting lump sum benefit in final salary pay plan (No.4 v.s.No.5)



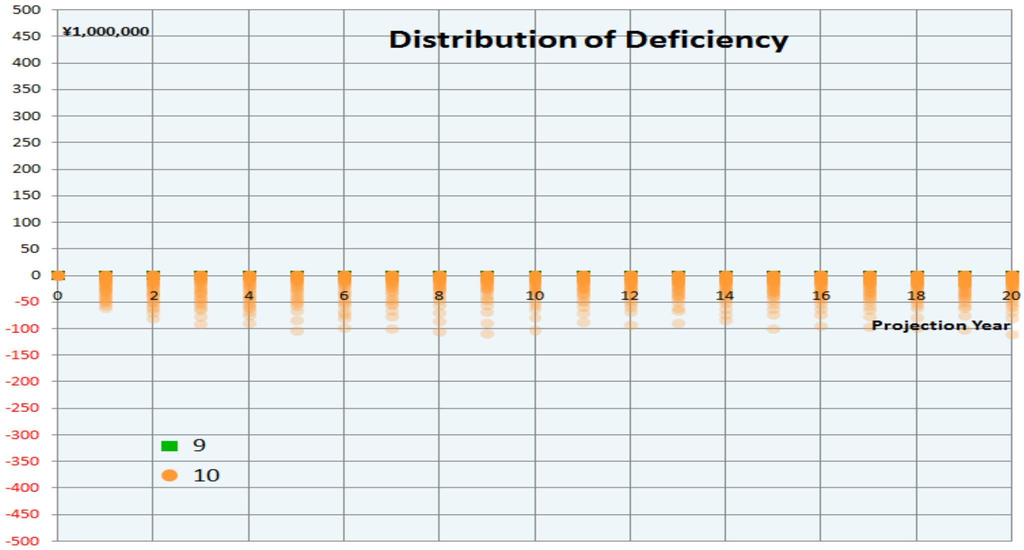
- By benefit formula (No.6 v.s.No.7)



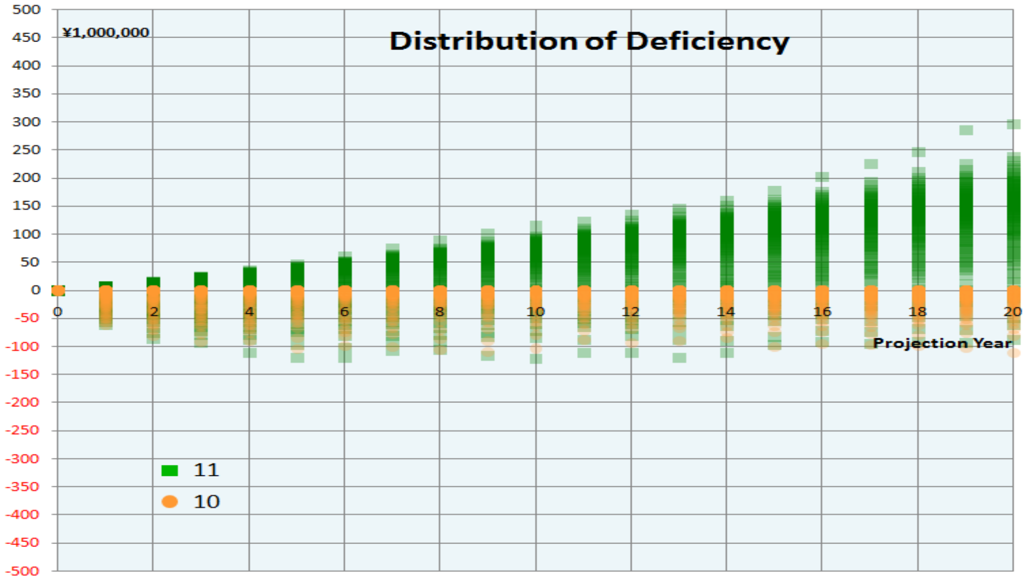
- By salary increase in CB plan (No.7 v.s.No.8), By actual return rate in CB plan (No.7 v.s.No.9)
  By withdrawal rate in CB plan (No.7 v.s.No.14,15)



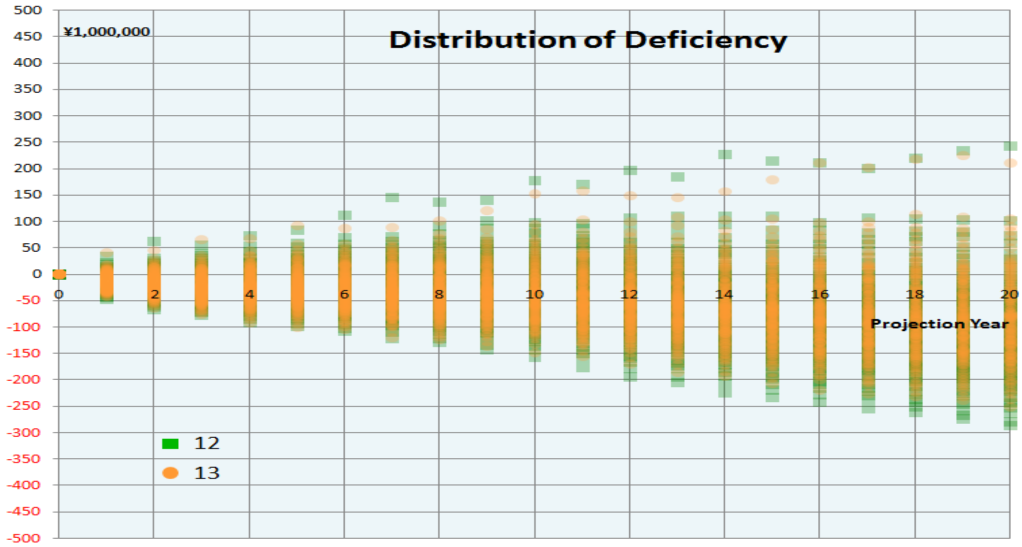
- By minimum guaranteed interest rate in CB plan (No.9 v.s.No.10)



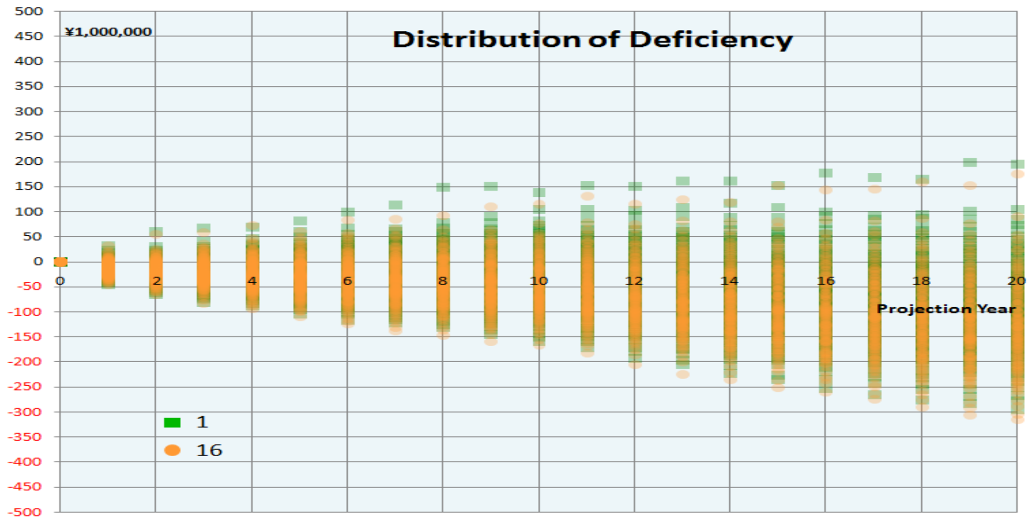
- By accumulated rate in CB plan (No.10 v.s.No.11)



- By withdrawal rate in final salary pay plan (No.12 v.s.No.13)



● By withdrawal rate in final salary pay plan (No.1 v.s.No.16)



In addition it is possible to show the distribution of desirable contribution rate as follows.

$$p^{Desirable} = Max(P^{Level}, P^{Min})$$

such that

p^{Level} : Level contribution rates.

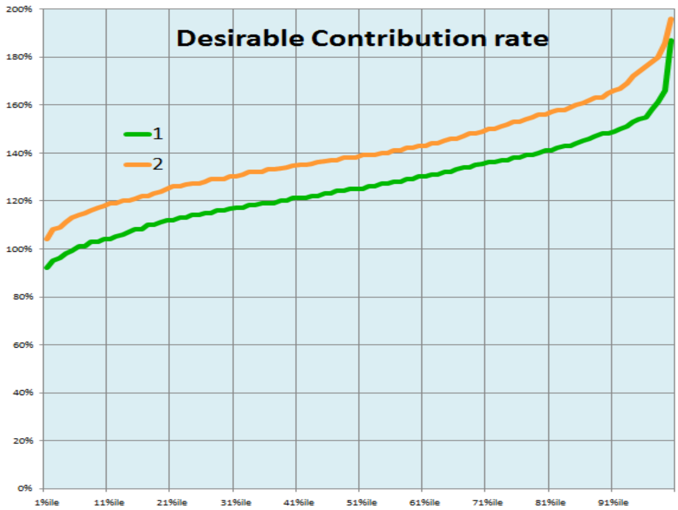
$$p^{Level} = \frac{Terminal\ value\ of\ (benefits - Assets)}{Terminal\ value\ of\ Salaries}$$

p^{Min} : Minimum contribution rates which lead no deficiency after certain years, for example 5 years.

$$p^{Min} = contribution\ rate\ which\ is\ already\ set + \frac{Deficiency\ at\ end\ of\ 5th\ projection\ year}{Terminal\ value\ of\ salaries\ at\ end\ of\ 5th\ projection\ year}$$

【Examples about desirable contribution rate】

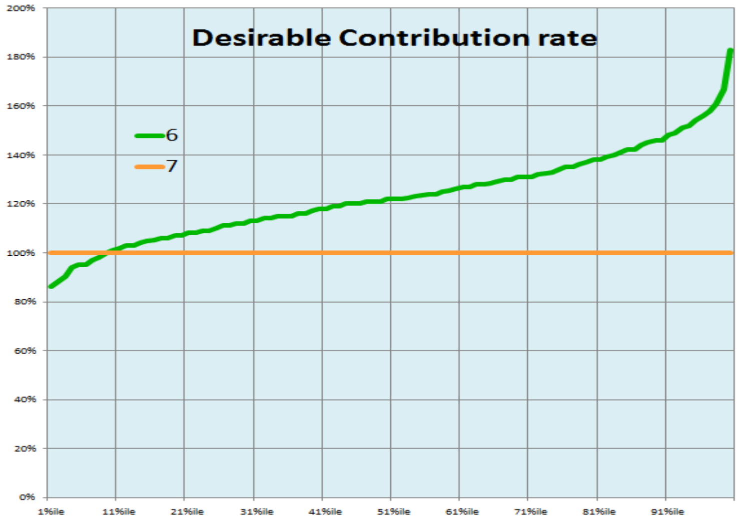
● By the salary increase in final salary pay plan (No.1 v.s.No.2)



Desirable = Max(①, ②)

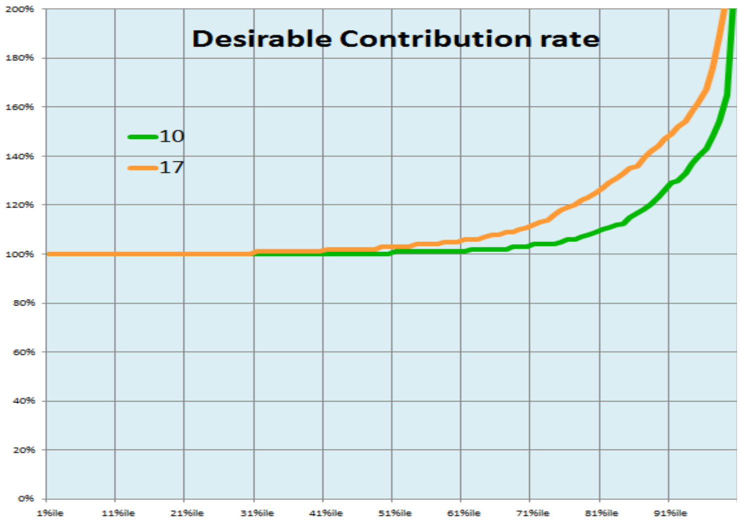
	case	contribution rate ①Pmin	contribution rate ②Plevel	MAX (①,②)
Average	1	124%	110%	126%
	2	138%	121%	140%
	1 − 2	**−15%**	**−11%**	**−14%**
Standard Deviation	1	19%	13%	17%
	2	20%	12%	18%
	1 − 2	**−1%**	**1%**	**−1%**
Max10%	1	148%	126%	148%
	2	165%	137%	165%
	1 − 2	**−16%**	**−11%**	**−17%**
Min10%	1	100%	93%	104%
	2	113%	106%	118%
	1 − 2	**−14%**	**−13%**	**−14%**

- By the benefit formula (No.6 v.s.No.7)



	case	contribution rate ①Pmin	②Plevel	MAX (①,②)
Average	6	120%	106%	123%
	7	100%	100%	100%
	6 － 7	**20%**	**6%**	**23%**
Standard Deviation	6	20%	13%	18%
	7	0%	0%	0%
	6 － 7	**20%**	**13%**	**18%**
Max10%	6	147%	123%	146%
	7	100%	100%	100%
	6 － 7	**47%**	**23%**	**46%**
Min10%	6	94%	90%	101%
	7	100%	100%	100%
	6 － 7	**−6%**	**−10%**	**1%**

- By the σ in CB plan (No.10 v.s.No.17)



	case	contribution rate ①Pmin	②Plevel	MAX (①,②)
Average	10	106%	102%	107%
	17	113%	104%	114%
	10 － 17	**−6%**	**−3%**	**−7%**
Standard Deviation	10	14%	3%	14%
	17	24%	7%	24%
	10 － 17	**−10%**	**−4%**	**−10%**
Max10%	10	126%	105%	126%
	17	147%	112%	147%
	10 － 17	**−21%**	**−7%**	**−21%**
Min10%	10	100%	100%	100%
	17	100%	100%	100%
	10 － 17	**0%**	**0%**	**0%**

Conclusion

There are several ways to show the results of simulations like above examples, which are not so difficult. However, at all events, it is important to show the results clearly so that plan sponsors can understand. In addition it is also important to show the result of simulation in which plan sponsors have interest. It is necessary for pension actuaries not only to check the finance of DB plans but also to explain it to them clearly.

I am deeply grateful to everyone who helped me making this paper.

Appendix

Assumption on Simulation

・Benefit formula

・Two types of benefit formulas of DB plans. One is based on final salary pay plan and the other is based on CB (Cash Balance) plan.

・DB plans pay benefits as all part of the lump-sum severance payment plans and provide annuity certain only which is familiar in Japan. Assume that period of pension payment is 10 years. ・Retirement age is 60 years old and pensions of DB plan start to be paid at 60 years old.

・The multiple $\kappa(t)$ in final salary pay plan are is defined as follows.

$\kappa(0) = 0$

$\kappa(t) - \kappa(t-1) = 0.5 \quad (1 \le t \le 10) \quad t: service\ years$

$\kappa(t) - \kappa(t-1) = 1.0 \quad (11 \le t \le 20)$

$\kappa(t) - \kappa(t-1) = 1.5 \quad (21 \le t \le 30)$

$\kappa(t) - \kappa(t-1) = 1.0 \quad (31 \le t \le 40)$

$\kappa(t) - \kappa(t-1) = 0 \quad (41 \le t)$

・Reduction ratio in voluntary withdrawal is defined as "$0.05 \times Min(20, t)$" or always 1 such that t is service years.

・Interest rate in replacing lump sum benefit with pension benefit and deferred interest rate are both 2.0%

・CB plan accumulates certain % of salary with actual return rate, which are not allowed yet in DB Law in our country. It is also assumed to set minimum guaranteed interest rate and/or certain rate to deduct from actual return rate as follows.

・Minimum guaranteed interest rate: 1%

・Certain rate to deduct from actual return rate: 1%

・Funding ratio for plan MFs

・Contribution accumulated in past service are equal to MF at the starting point (i.e. t=0) of simulation1. Plan assets are equal to plan MFs at the starting point (i.e. t=0) of simulation2.

・Calculation of Contribution is based on Entry Age Normal Cost Method with above assumptions which are belong to "No.1" except assumed interest rate.

・Actual return on assets investment portfolio

Expected actual return rate on plan assets μ is 2%, 3% and expected standard deviation σ is 2% , 5% each in Simulation2. In addition σ is 7% in case of lower side stress test.

・Timing of benefit payment and contribution accumulated

Contributions are paid at beginning of the year. Lump sum benefits are paid immediately to withdrawal participants at end of the year.

・Actuarial assumptions about the participants of DB plan is as follows.

・Participants are all male and do not die. (There is almost no problem to simulate in annuity certain.)

・Current ages of participants are 18 years old to 59 years old and past services are 0 years to 41 years in Simulation1.

In Simulation2 there are 80 participants and no pensioners at the starting point (i.e. t=0) of simulation. All of them entered at 20 years old and every two of them are same ages which are 20 to 59 years old.

・The birthdays of participants are all beginning of the year.

・Probability of withdrawal in Similation2 is based on experienced withdrawal rate of small TQPPs at each age and multiplying those rate by certain rate based on 95% credibility interval in stress test.(*7)

・Participants can select pensions replacing with a lump sum benefits if their service years are 20 or more at withdrawal though they do not select partial replacing in Simulation2.

・Range of participant's salary are defined as "$20+Max(0,Min(35,x-20))\times r \quad 0.6\le r\le 1.4$" such that x is current or future age in final salary pay plan. In case of higher side stress test, $0.9\le r\le 1.7$.

In CB plan those range are defined so that the benefits for participant who enters at 20 years old are same as that of final salary pay plan.

・Actuarial assumptions used in calculating contribution

・Assumed interest rate : 2% or 3%

・withdrawal rates : experienced withdrawal rate of small TQPPs

・ratio that participants select pensions replacing with a lump sum benefits : 100%

・Salary increase rates : described above in case that $r=0$.

<u>Formula of Simulation2</u>

● $P(x,t)$: t-th year's contribution for plan participant [x] who is age x at starting point $(1\le t\le y-x)$

y:retirement age

$$P(x,t)=\pi(t)\times S(x,t-1)\times L(x,t-1) \quad ;t\ge 1$$

$\pi(t)$:t-th year's annual contribution rate

$S(x,t)$: random variable which expresses the salary of plan participant [x] t years later.

$L(x,t)$: random variable which expresses the condition of plan participant [x] t years later, which value is 1 in case of employment and 0 in case of post-employment.

● $K^0(x,t,j)$: t -th year's lump-sum benefit for withdrawal reason j for plan participant [x] $(t\le y-x)$

$$K^0(x,t,j)=S^K(x,t)\times k(x,t_0+t,j)\times\{1-L(x,t)\}\times\{1-c(x,t)\}\times H(t)$$

such that $S^K(x,t)=S(x,t)$ ・・・Final salary pay plan

$=Max(\sum_{s=0}^{t-1}[S(x,s)\prod_{u=s+1}^{t}\{1+i(u)-i^k\}],\sum_{s=0}^{t-1}\{S(x,s)\times(1+i^g)^{t-s}\})$ ・・・CB plan

such that $i(t)$: t-th year's random variable which expresses actual return rate

i^g : minimum guaranteed interest rate

i^k : certain rate to deduct from actual return rate

$k(x,t_0+t,j)$: t -th year's lump-sum benefit ratio for withdrawal reason j for plan participant [x] with past services (t_0).

$c(x,t)$: random variable which expresses the condition whether plan participant [x] chooses lump-sum benefit t years later, which value is 0 if choosing it and 1 if not choosing it.

$H(t)$: 1 if $L(x,t-1)$ is 1 and $L(x,t)$ is 0. 0 if otherwise.

● $F(x,t,j)$:deferred pension fund t years later for withdrawal reason j for plan participant [x]. $(t\le y-x)$

・Final salary pay plan

$$F(x,t,j)=\sum_{s=1}^{t} S^K(x,s)\times k(x,t_0+s,j)\times\{1-L(x,s)\}\times c(x,s)\times H(s)\times(1+i^d)^{t-s}$$

such that i^d is deferred interest rate.

・CB plan

$$F(x,t,j)=\sum_{s=1}^{t} S^K(x,s)\times k(x,t_0+s,j)\times\{1-L(x,s)\}\times c(x,s)\times H(s)\times i^D(s,t)$$

such that $i^D(s,t)=\prod_{u=s+1}^{t}\{1+i(u)\}$.

Remark: Assume that plan does not guarantee minimum interest rate in post-retirement.

● $K^1(x,t,j)$: t -th year's pension benefit for withdrawal reason j for plan participant [x]

・Final salary pay plan

$$K^1(x,t,j)=\frac{F(x,y-x,j)}{\ddot{a}^{(i^a)}_{\overline{n}|}} \quad (1\le y-x\le t\le y-x+n-1)$$ n : period of pension payment(=10 years)

$= 0$ (otherwise)

such that i^a is the interest rate in calculating an amount of pension when plan participant chooses pension in place of lump sum benefit.

・CB plan

$$K^1(x,y-x,j)=\frac{F(x,y-x,j)}{\ddot{a}^{(i(y-x))}_{\overline{n}|}}$$

$$K^1(x,t,j)=\frac{\{F(x,t-1,j)-K^1(x,t-1,j)\}\times\{1+i(t)\}}{\ddot{a}^{(i(t))}_{\overline{n-\{t-(y-x)\}}|}} \quad (y-x<t\le y-x+n-1)$$

$= 0$ (otherwise)

● $F(x,t,j)$: pension fund t years later for withdrawal reason j for plan participant [x].

$(y-x<t\le y-x+n-1)$

・Final salary pay plan

$$F(x,t,j)=\{F(x,t-1,j)-K^1(x,t-1,j)\}\times(1+i^a)$$

= 0 (otherwise)

・CB plan

$$F(x,t,j)=\{F(x,t-1,j)-K^1(x,t-1,j)\}\times\{1+i(t)\}$$

= 0 (otherwise)

● $A(T)$: plan assets T years later $(0\le T\le 55)$

$$A(0)=A_0$$

$$A(T+1)=\{A(T)+\sum_x P(x,T+1)-\sum_x K^1(x,T+1,j)\}\times\{1+i(T+1)\} \quad -\sum_x K^0(x,T+1,j)$$

● $MF(T)$: plan MFs T years later

$$MF(T)=\sum_{[x]}MF^{[x]}(T)+\sum_{[x]}F(x,T,j) \qquad (x+T\le y)$$
$$=\sum_{[x]}F(x,T,j) \qquad (x+T>y)$$

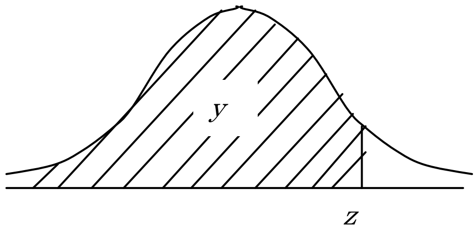
such that $MF^{[x]}(T)=S^K(x,T)\times k(x,T,j)\times L(x,T)$

¶ About above random variables

・ $i(t)$

Let $i(t)$ be a normal distributed random variable with mean μ and standard deviation σ and the value is same at t for all formula with all plan participants.
It is possible to get $i(t)$ on PC in following way.

$i(t)=\{z\,|\int_{-1}^{z}\frac{1}{\sqrt{2\pi}\sigma}e^{-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2}dx=y\}$ such that y is uniformly random variable on [0,1]



Remark : It is no problem to assume $i(t)>-1$

Other random variables are as follows.

・ $S(x,t)$

$$S(x,t)=Max(b^S_{x+t},Min(b^L_{x+t},b^S_{x+t}+\frac{\beta(x,t-1)-b^S_{x+t-1}}{b^L_{x+t-1}-b^S_{x+t-1}}(b^L_{x+t}-b^S_{x+t}))) \qquad (t>0)$$

= salary of participant' data used in this simulation $(t=0)$

such that $\beta(x,t-1)$ is assumed to be a normal distributed random variable with $\mu=S(x,t-1)$ and $\sigma=\frac{b^L_{x+t-1}-b^S_{x+t-1}}{2\times 1.96}$ (*).

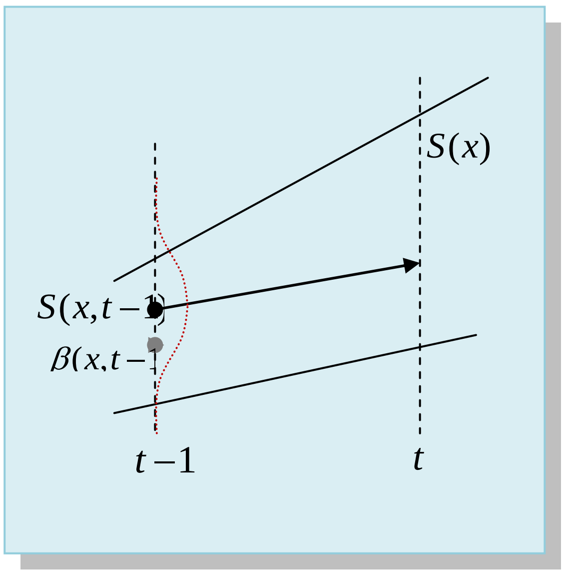
However $\beta(x,t-1)$ is also assumed to be $b^S{}_{x+t-1}$ if it is smaller than $b^S{}_{x+t-1}$ and $b^L{}_{x+t-1}$ if it is larger than $b^L{}_{x+t-1}$

$b^S{}_{x+t}$: minimum salary of participant of age (x+t)
$20+Max(0,Min(35,x-20))\times 0.6\ or\ 0.9$

$b^L{}_{x+t}$: maximum salary of participant of age (x+t)
$20+Max(0,Min(35,x-20))\times 1.4\ or\ 1.7$

The setting mentioned above shows that the salary of participant of age (x+t) is determined using normal distribution based on where the salary of last year exists in the rage between maximum salary and minimum salary.
In case of final salary pay plan, $S(x,t)$ is replaced with $S(x,t-1)$ if $x>55$ or it is smaller than $S(x,t-1)$ after above calculations.

$S(x,t)$ =25,23,22,29,27…→ 25,25,25,29,29…



・ $L(x,t)$

$$L(x,t) = \prod_{s=1}^{t} \omega(x,s)$$

Such that $\omega(x,s) = 0$ if $z \le \theta_{x+s-1}$ which z is an uniform distributed random variable on [0,1]

$=1$ if $z > \theta_{x+s-1}$ which z is an uniform distributed random variable on [0,1]

θ_{x+s-1} : experienced withdrawal rate of small TQPPs at age ($x+s-1$)

Especially $L(x,0)=1$

・ $c(x,t)$

$c(x,t)=0$ if $z \le c_0$ which z is an uniform distributed random variable on [0,1]

$=1$ if $z > c_0$ which z is an uniform distributed random variable on [0,1]

c_0 : experienced ratio which participants choose lump sum benefits

Especially $c(x,0)=0$

The Monte Carlo simulation based on above formulas gives distribution of plan deficiency below MFs in the future, which uses small DB plan which consists of 80 participants in above examples.

In addition assume that all random variables are mutually independent though they may be correlated.

(*)Assume that paticipant's salary almost fluctuates in width of $b_{x+t-1}^{L} - b_{x+t-1}^{S}$ with probability of 95%.

Notes: (References)

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